APPENDIX 5 – DERIVATION OF EQUATIONS 6-1 AND 6-2

I derived these equations to determine the power transmitted through a transmission line with attenuation and terminated in a mismatch (see Fig Appendix 5-1).

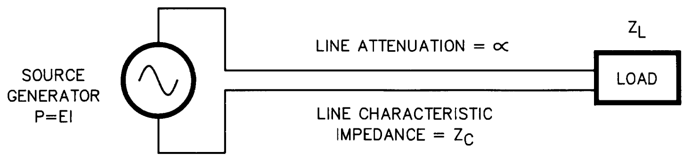

Fig Appendix 5-1 – Illustrating the transmission line for the Appendix 5 derivation of Eqs 6-1 and 6-2.

ρ_L = voltage reflection coefficient at load

$$= \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{SWR - 1}{SWR + 1} \quad \text{(Eqs 3-1 and 3-2)}$$

ρ_I at line input = $\rho\varepsilon^{-2\alpha}$

ρ_I^2 at line input = $\rho^2\varepsilon^{-4\alpha}$

ρ_L^2 = power reflection coefficient at load (equals reflected power)

where

$$\alpha = \text{line attenuation in nepers} = \frac{dB}{20 \log \varepsilon} = \frac{dB}{8.6859}$$

(one neper = 0.115129 decibel)

ε = 2.718282
(the base of natural logarithms)

The forward *voltage* wave encounters a reduction in level by the loss factor of $1/\varepsilon^{\alpha}$, or $\varepsilon^{-\alpha}$ in traveling from the source to the load. The reflected voltage wave encounters the same reduction factor $\varepsilon^{-\alpha}$ in its return to the source. Hence, the voltage reflection coefficient at the line input is $\rho_I = \rho\varepsilon^{-2\alpha}$.

The forward *power* encounters a *one-way* attenuation-loss factor ε^{-2a}. Power returning to the source from a load reflection encounters a *two-way* loss — once during the trip to the load, and again during its return to the source. The power reflection coefficient at the line input is therefore $\rho_I^2 = \rho^2\varepsilon^{-4\alpha}$.

Let power supplied by the source = 1, and let reflected power reaching the input (which adds to the source power) = r. Then,

1 + r = total forward power at line input,

$(1 + r)\varepsilon^{-2\alpha}$ = forward power arriving at the load, and

$\frac{r}{1+r}$ = input power reflection coefficient = $\rho_I^2 = \rho^2\varepsilon^{-4\alpha}$.

Now let $\frac{r}{1+r}$ (and $\rho^2\varepsilon^{-4\alpha}$) = a.

Then, total forward power (1 + r) at the line input = $1+\frac{a}{1-a} = \frac{1-a+a}{1-a} = \frac{1}{1-a}$.

Hence, it follows that

Forward power at line input		Forward power at load after line atten.		Power reflected at load		Power absorbed in load
$1+r$	→	$(1+r)\varepsilon^{-2\alpha}$	–	$\rho^2(1+r)\varepsilon^{-2\alpha}$	=	$(1+\rho^2)(1+r)\varepsilon^{-2\alpha}$

and

$$\frac{1}{1-a} \rightarrow \frac{\varepsilon^{-2\alpha}}{1-a} - \frac{\rho^2\varepsilon^{-2\alpha}}{1-a} = \frac{(1-\rho^2)\varepsilon^{-2\alpha}}{1-a}$$

But $a = \rho^2\varepsilon^{-4\alpha}$, so by substitution we have Eq 6-1:

$$\frac{1}{1-\rho^2\varepsilon^{-4\alpha}} \rightarrow \frac{\varepsilon^{-2\alpha}}{1-\rho^2\varepsilon^{-4\alpha}} - \frac{\rho^2\varepsilon^{-2\alpha}}{1-\rho^2\varepsilon^{-4\alpha}} = \frac{(1-\rho^2)\varepsilon^{-2\alpha}}{1-\rho^2\varepsilon^{-4\alpha}}.$$

Translating the last expression on the right, it means that power absorbed in the load equals the source power times the quantity

$$\frac{(1-\rho^2) \text{ times one – way line – attenuation factor}}{1-(\rho^2 \text{ time two – way line – attenuation factor})}$$

where $\rho = \rho_L$ from Eqs 3-1 and 3-2 above.

$$\frac{1}{1-\rho^2} \rightarrow \frac{1}{1-\rho^2} - \frac{\rho^2}{1-\rho^2} = 1$$

$$1+r \rightarrow 1+r - r = 1$$

When the SWR on a transmission line is ≈ 4.6:1 the total attenuation α_T is approximately two-times the attenuation of the line when matched.