APPENDIX 7 – ON THE INCREASE IN FORWARD POWER RESULTING FROM CONJUGATE MATCHING AT THE INPUT OF A MISMATCHED TRANS MISSION LINE

(A calculator program for solutions using this procedure appears in Appendix 7a.)

When using coaxial transmission lines with a mismatched termination that gives rise to standing waves on the line, we must be concerned with the limitation that must be imposed on the magnitude of the standing waves by the voltage and power handling capability of the line. Thus, we must find a way to determine the amount of power traveling on the line with respect to the magnitude of the standing waves.

It is well known that when a transmission line has a mismatched termination, but which is conjugately matched at the input of the line, the forward power propagating along the line is greater than the source power by the amount of the reflected power. Consequently, on lossless lines, the power absorbed by the mismatched termination equals the total forward power minus the reflected power. However, for this phenomenon to occur the power traveling forward in the line must exceed that supplied by the generator. How can this be, you ask? The following discussion will explain this phenomenon *(Ref 18, Page 99)*.

When a wave of energy is first applied to the line, it sees only the characteristic impedance Z_0 in its travel along the line until it arrives at a mismatched termination. On arriving at the mismatch the wave is reflected with the reflection coefficient magnitude

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0},$$

where
ρ = reflection coefficient of either voltage or current
Z_0 = characteristic impedance of the line
Z_L = impedance of the line termination.

The reflection at the mismatch gives rise to a second wave that travels back to the input of the line with the magnitude determined by multiplying the magnitude of the initial wave by the magnitude ρ. On arriving at the line input it sees total reflection $\rho = 1$ at the matching point in the network that achieved the conjugate match. Thus reflection of the second wave gives rise to a third wave of the same magnitude as that of the second wave, which also is reflected by the mismatched termination. This reflection thus gives rise to a fourth wave with the magnitude determined by multiplying the magnitude of the third wave by the magnitude ρ.

This process continues indefinitely, with each reflection smaller than the last. We thus have an infinite series of reflections, whose sum is convergent and represents the steady state condition, which is finally established. The infinite series can be written in the simplified form as $1 + a + a^2 + a^3\ldots$, which converges to the value

$$\frac{1}{1-a}, \text{ where } |a| < 1. \qquad \text{Eq (1)}$$

We use this equation of convergence in solving for the total forward power in the line by letting 'a' equal the *power reflection coefficient* ρ^2. Because the power absorbed in the load equals the forward power minus the power reflected, the *power transmission coefficient* becomes $1 - \rho^2$. We now replace 'a' with ρ^2 in the convergence equation to get

$$\frac{1}{1-\rho^2}, \qquad \text{Eq (2)}$$

which represents the sum of the infinite number of reflections appearing on a mismatched line when conjugately matched at the line input. Inserting the value of ρ^2 in the equation gives us the number to multiply by the source power to determine the total power propagating in the forward direction toward the mismatched load, when the line is conjugately matched at the input with a matching network, such as an antenna tuner. The numerical result from calculating Eq (2) is called the *forward power increase factor.*

Let us illustrate this principle with an example. We will use the same values in the example that appear in the examples cited in Chapters 3 and 9, where the characteristic impedance of the transmission line is $Z_0 = 50$ ohms, and the mismatched load is a 150-ohm resistance, yielding a 3:1 SWR. The reflection coefficient $\rho = 0.5$, and therefore the power reflection coefficient $\rho^2 = 0.25$. Substituting 0.25 in the convergence equation, we get

$$\frac{1}{1-0.25} = \frac{1}{0.75} = 1.3333$$

Thus, the forward power increase factor in this example is 1.3333. Consequently, if we supply 100 watts to the line with a 3:1mismatch, but conjugately matched to the RF generator at the input of the line, we find the total forward power to be 100 watts $\times$ 1.3333 = 133.33 watts

Continuing, let us determine the actual power incident on the mismatched load, the power absorbed by the load and the power reflected by the load. Recall that the power *transmission* coefficient $(1 - \rho^2) = 0.75$, which means that 75% of the power incident on the mismatched load will be *absorbed*. Also recall that the power *reflection* coefficient $\rho^2 = 0.25$, which means that 25% of the power incident on the load will be *reflected*. Multiplying out these percentages shows that 133.33 watts $\times$ 0.75 = 100 watts absorbed, and that 133.33 watts × 0.25 = 33.33 watts reflected. If we now add the 33.33 watts of reflected power to the 100 watts absorbed in the mismatched load we get 133.33 watts, thus proving that we have conformed to the Law of Conservation of Energy. We have also shown that, with lossless line, where there is no loss due to line attenuation, all the power delivered by the RF generator is absorbed in the load, even though mismatched 3:1 to the transmission line delivering the power.

Let us now apply the procedure to real transmission lines having loss due to attenuation. We perform this procedure by reducing the value of of the reflection coefficient ρ by the amount of line attenuation α in decibels. We first convert the attenuation in decibels to its decimal value, and then multiply ρ by the decimal value to obtain the modified value of ρ that includes the effect of the line attenuation. The decimal number α_{DEC} of α is obtained from the expression

$$\alpha_{\mathrm{DEC}} = \text{anti}\log_{10}^{-1}\frac{\alpha}{10}. \qquad \text{Eq (3)}$$

We will now illustrate the procedure with an example. We will use the example above where the mismatch yields reflection coefficient ρ = 0.5, but where the line attenuation α = 0.5 dB. Dividing 0.5 by 10 yields 0.05. The expression 'anti log–1' means taking the reciprocal of the number obtained by evaluating the number $10^{0.05}$, by raising 10 (the base of the logarithm) to the power 0.05. The number α_{DEC} can also be found by evaluating 10 with exponent –0.05, ($10^{-0.05}$) without taking the reciprocal.

Continuing the example, α_{DEC} = the reciprocal of $10^{0.05}$ = 0.89125, the decimal value of 0.5 dB. ($10^{-0.05}$ also equals 0.89125.) Now we multiply ρ by α_{DEC} to obtain the new value of ρ' that includes the effect of the line attenuation.

ρ' = 0.5 × 0.89125 = 0.44563

Squaring ρ' to obtain the new power reflection coefficient, $\rho'^2 = 0.44563^2 = 0.19858$. Replacing ρ^2 with $\rho'^2 = 0.19858$ in Eq (2) yields

$$\frac{1}{1-0.19858} = \frac{1}{0.80142} = 1.2478.$$

Thus, the forward power increase factor in this example is 1.2478. And as in the first example, if we supply 100 watts to the line having a 3:1 mismatch, conjugately matched to the RF generator at the input of the line, but with the line now having a 0.5 dB attenuation, we find the forward power *at the input of the line* to be 100 watts × 1.2478 = 124.78 watts. However, due to the 0.5 dB line attenuation, the forward power encounters a 0.5 dB loss during its travel to the load, leaving 111.21 watts incident on the load. To confirm the incident power of 111.21 watts after 0.5 dB attenuation from the 124.78 watts at the line input, we see that

$$\frac{111.21\text{ watts}}{124.78\text{ watts}} = 0.89125,$$

which is the decimal number α_{DEC} that equals 0.5 dB obtained from the calculations above.

A diagram of this example, and further explanation from a different viewpoint appears in Fig Appendix 6-4B and *Example 5*.

APPENDIX 7B – CALCULATOR PROGRAM FOR DETERMINING THE INCREASE IN FORWARD POWER RESULTING FROM CONJUGATE MATCHING AT THE INPUT OF A MISMATCHED TRANSMISSION LINE

Listed below is a calculator program for determining the increase in forward power resulting from conjugate matching at the input of a mismatched transmission line, using the procedure described in Appendix 7. This listing is for a programmable Hewlett-Packard calculator, or other brand using reverse Polish notation.

Line	*Key Entry*	*Comments*	*Instructions*
1	H Label A		1. Store line attenuation in Register R_1
2	1		2. Store SWR in Register R_2
3	–		3. RUN
4	RCL 2	SWR	
5	1		
6	+		
7	÷		
8	STO 8		
9	R/S		
10	$g \rightarrow x^2$	ρ^2	
11	1		
12	↔		
13	–		
14	STO 5		
15	R/S	$(1 - \rho^2)$	
16	RCL 1	α in dB	
17	CHS		
18	ENTER		
19	1		
20	0		
21	÷		
22	$g \rightarrow 10^X$		
23	STO 9		
24	R/S	α_{DEC}	
25	RCL 8	ρ	
26	×	ρ'	
27	$g \rightarrow x^2$	ρ'^2	
28	1		
29	↔		
30	–		
31	STO4		

32	R/S	$(1 - \rho'^2)$
33	1	
34	↔	
35	÷	
36	H Return	