Chapter 3

Going Around in Circles to Get to the Point

(Adapted from *QST*, August 1973)

Sec 3.1 Basic Reflection Mechanics and Wave Propagation

In this chapter, I discuss traveling waves and their reflections on transmission lines, how they are developed, and how they both effect and affect operations on the lines. In describing the direction of wave travel, the terms "forward" and "incident" are used interchangeably.

It is well known that the size, or magnitude, of a reflection arising from a mismatched line termination is determined by the degree of the mismatch, or the amount of power in the forward, or incident wave that is not absorbed by the load terminating the line. In this book the symbol ρ (rho) represents the *magnitude* of a reflection, called the *reflection coefficient,* which expresses the ratio of the voltage reflected by the mismatch relative to the forward voltage, the voltage incident on the load.

However, for a complete description of the reflection, in addition to its magnitude we must also consider its *phase.* The phase of the mismatch reflection, represented by the symbol θ (theta), is the phase angle of the reflected voltage relative to that of the forward voltage at any given point on the line. In combination, the magnitude ratio ρ, and phase θ, comprise the *complex* reflection coefficient $\overline{\rho} = \rho\angle\theta$, which becomes important in later discussions of transmission-line wave propagation and impedance matching techniques. The complex coefficient, $\overline{\rho}$, is determined quantitatively from the line and load impedances by the expression

$$\overline{\rho} = \frac{Z_L - Z_C}{Z_L + Z_C} \qquad \text{(Eq 3-1)}$$

where

Z_L = the complex load impedance, R + *j*X

Z_C = the characteristic impedance of the transmission line.

This expression shows that $\overline{\rho} = 0$ (no reflection) when $Z_L = Z_C$. However, for zero reflection, the load must be purely resistive (R + *j*0), because here we are considering only lossless and low-loss lines with a characteristic impedance Z_C that is real, that is, Z_C = Re(V/I) + Im(0). (Re is the dissipationless, *real part* of impedance.)

For those of you who are wondering why I use ρ instead of Γ to represent reflection coefficient, I digress here to present a note of historical significance that will answer your question. Prior to the 1950's ρ (rho), σ (sigma), and sometimes S, were used to represent standing-wave ratio. The symbol of choice to represent reflection coefficient during that era was Γ, upper case gamma. However, in 1953 the American Standards Association (now the NTIA) announced in its publication ASA Y10.9-1953, that ρ is to replace Γ as the standard symbol for reflection coefficient, with SWR to represent standing-wave ratio (for either voltage or current), and VSWR specifically for *voltage* standing-wave ratio. Most of academia responded to the change, but some individuals did not. Consequently, Γ is occasionally seen representing reflection coefficient, but only rarely.

Before delving into the details of the reflection mechanics of wave reflections on transmission lines, it will be helpful in understanding the mechanics if we first examine the origination of waves on transmission lines, and how they travel on the line. We'll begin by introducing an ac voltage across the input terminals of the line, which sets up an electric field between the conductors of the line. Because the voltage introduced into the line is ac the resulting electric field is constantly changing. At this point an *electromagnetic* field is developing, because a changing electric field produces a changing magnetic field. In turn, the changing magnetic field causes current flow on the conductors. The changing magnetic field also produces a changing electric field, which produces a magnetic field, and so on indefinitely, until the voltage at the input of the line is cut off. However, with the continuing alternate production of the electric and magnetic fields a series of energy transfers is started. The magnetic energy does not appear at precisely the same point on the line as the electric energy from which it is derived, but a little beyond, and then the succeeding electric energy is formed still a little farther down the line, so that as the energy is changing from form to form it is also being propagated along the line. The result is a traveling wave of electromagnetic energy propagating along the line. It is therefore evident that if there is a changing electric field there must also be a changing magnetic field. Thus the name *electromagnetic* field — if it exists it must contain both electric and magnetic fields simultaneously.

To digress from the discussion of propagation on transmission lines for a moment, it is also of interest to know that the same phenomenon occurs in space as just described for the transmission line. The electromagnetic field radiated from an antenna propagates through space in precisely the same manner. Although we didn't discuss it above, it should be known that the electric and magnetic fields are always at right angles or perpendicular to each other, and in directions that are also perpendicular to the direction of the wave travel. This geometric relationship holds true for the propagation in transmission lines, as well as in space. Consequently, we can define polarization of the electromagnetic field. It is universally accepted that the polarization is determined by the orientation of the *electric* field. When the electric field is vertical, the polarization is said to be vertical, and when the electric field is horizontal, the polarization is horizontal. Therefore, you know that when your antenna elements are oriented in the horizontal position, and receiving the maximum signal with respect to orientation of the elements, the incoming wave is horizontally polarized, and the magnetic field is vertical.

We are now ready to return to the discussion of reflections and reflection coefficients. We begin by first describing the nature of reflections, and how they are produced. An open circuit (an infinite impedance), a short circuit (zero impedance), or a pure reactance terminating a transmission line is incapable of absorbing any power from a forward wave, and therefore causes total reflection of the energy in the forward waves of both voltage and current. The magnitude ρ of the reflection coefficient at such a load impedance is therefore 1.0 (unity) for both voltage and current.

The step-by-step process by which the reflections arise on lossless and low-loss transmission lines, both coaxial and open-wire, is as follows, beginning with the manner in which waves of voltage and current on transmission lines are formed initially. If a sinusoidal alternating voltage is applied across the input terminals of a transmission line, a forward-traveling voltage wave is launched into the line traveling toward the load. Simultaneously a forward-traveling current wave is launched into the line, accompanying the voltage wave in its travel toward the load. Together, the voltage and current waves develop a forward-traveling electromagnetic field comprising an electric field and a magnetic field. One-half of the energy in the electromagnetic field is stored in the electric field, because of forward voltage, and the other half is stored in the magnetic field, because of forward current. The input impedance that the electromagnetic field encounters as it leaves the generator in its forward travel down the transmission line is the *surge* impedance of the line, also known as the *characteristic* impedance, Z_C . In ideal lossless lines the voltage and current travel in perfect phase with each other because the characteristic impedance Z_C is real, since $Z_C = \text{Re}\ (V/I) + \text{Im}\ (0)$, as stated earlier. However, in real lines of low-loss, such as those generally used in amateur and commercial RF practice, there is a small negative reactance, (capacitive, $-X_C$), in the characteristic impedance, but it is so small that it can usually be neglected. Therefore, the voltage and current in real lines are essentially in phase during travel along the line. On the other hand, the capacitive reactance appearing in lossy lines cannot be neglected, and must be taken into consideration when lossy lines are used in engineering applications.

Now we'll proceed to the generation of reflections. When the electromagnetic field reaches the end of the line, if the load terminating the line is an open circuit, the magnetic field collapses because the current goes to zero due to the infinite impedance of the open-circuit. The changing magnetic field at the open circuit produces a new electric field equal in energy to the magnetic field, which induces a new voltage into the load circuit that is equal to, and in phase with the voltage in the forward wave. (Keep in mind that a voltage is induced, or generated, by mutual motion between a magnetic field and a conductor, a phenomenon generally known as *motor-generator* action. Thus, it can be said that the reflected voltage was developed and delivered by a generator, a *reflection generator*. Although in this case the *field* is changing while the conductor is stationary, as in a transformer, it is motor-generator action nonetheless.) The new electric field induced by the changing magnetic field adds *in phase* to the existing electric field, and the new induced voltage (delivered by the reflection generator) adds *in phase* to the voltage in the forward wave, resulting in an increase of voltage at the open circuit to twice the voltage of the forward wave. At this instant, a standing wave is developing, because now there is a current minimum and a voltage maximum at the open-circuit termination, where an instant before, current and voltage were constant all along the line.

The new voltage at the open-circuit termination, along with its new electric field, starts a voltage wave traveling in the rearward direction, as if it had been launched by a *separate generator* at the open-circuit point. (It has — remember the induced voltage, *generated* by the changing magnetic field?) Since no energy was absorbed by the open-circuit load, the new rearward-traveling voltage wave has the same magnitude as the original forward wave, which is why $\rho = 1$, indicating total reflection. As the new electric field starts its rearward travel, it produces a new magnetic field, which in turn produces a new current, launched into the line as the reflected current wave with the same magnitude as the forward current wave, but with opposite polarity and direction. The new electric and magnetic fields combine to form the reflected electromagnetic field and, as in the forward electromagnetic-field wave, the energy in the reflected electromagnetic-field wave also divides equally between its electric and magnetic fields. *(Ref 17, p 139; Ref 19, p 4; Ref 35, p 21; Ref 43; Ref 70).*

To continue, the total voltage (or current) at the load at any instant is the sum of the voltages (or currents) of the forward and reflected waves. The in-phase reflected voltage wave is verified because the sum of the two voltages at the load is equal to two times that of the forward voltage. In addition, since the two current waves of opposite polarity add to zero at the open-circuit load, the generation of the reversed-polarity, out-of-phase reflected-current wave is also verified. The phase angles, θ, of the reflection coefficients at the open-circuit load are therefore 0° for voltage and 180° for current.

When the load impedance is a short circuit, the reflection-generation process is the same as the open-circuit process described above, except that the electric- and magnetic-field actions and the polarities of the reflected-wave components are reversed. This is expected when we recall that, while current goes to zero in an open circuit, voltage must be zero in a short circuit. For the voltage to be zero, the forward and reflected voltages must cancel one another at the load, thus verifying the reversed polarity of the reflected voltage with the short-circuit termination. The corresponding currents are of the same polarity, and add at the short-circuit load to equal twice the forward value, as the voltages did when the load was an open circuit. The phase angles θ, of the reflection coefficients at the short-circuit load are therefore 180° for voltage and 0° for current. When the load impedance is a pure capacitance, it is equivalent to an additional length of open-circuited line, while a purely inductive load is equivalent to an additional length of short-circuited line.

We have observed that on reflection at a mismatched load termination, either the voltage *or* the current changes polarity, ***but not both.*** Repeating for emphasis and clarity, when the load is an open circuit the current changes polarity, but not the voltage; when the load is a short circuit the voltage changes polarity, but not the current. Hence, it is evident that while the forward voltage and current travel in phase, the reflected voltage and current are 180° out of phase, and maintain that phase relationship during their entire rearward travel on the line.

When the load impedance contains resistance that is not equal to the characteristic impedance Z_C the reflection is generated in the same manner as with infinite- or zero-impedance loads described earlier, except that the reflection is less than total, the magnitude depending on the amount of power absorbed in the resistance. The reflected wave is again generated by the changing electric and magnetic fields in the mismatched resistance due to the change in voltage and current when the forward wave encounters a change in impedance relative to the characteristic impedance Z_C of the transmission line. Hence, the reflection coefficient ρ is dependent on the difference between the forward-wave voltage on the line and the resulting voltage appearing across the load. If the load contains no reactance, and its resistance is greater than Z_C, the reflection angles θ are the same as for the open-circuit termination; if the load resistance is less than Z_C, the θ angles are the same as for the short-circuit termination.

No reflection arises when the load is a pure resistance equal to Z_C, because all of the forward-traveling energy is absorbed in the load, and because there is no variation in voltage or current when the energy passes from the line to the load. Thus, there is no change in the electric and magnetic fields, no new voltage or current generated, and hence no reflected wave.

Returning now to the wave action on the line, after being launched rearward the reflected wave travels back up the line as a new electromagnetic traveling wave, separate and distinct from the forward-traveling incident wave. As the reflected wave travels rearward, it encounters only the same real line impedance Z_C encountered by the forward wave in its forward travel to the load. Hence, the magnitudes of both the reflected voltage and current waves remain constant as they plow rearward, retaining the same values as when leaving the reflection generator. (This is exactly true only on lossless lines. However, there is a *gradual* reduction in magnitudes with line length in real lines due to line attenuation, as discussed later.) The magnitudes of the reflected waves are completely unaffected by the standing wave that is developing as the reflected and forward waves slide past one another traveling in opposite directions. The forward voltage and current waves are similarly unaffected, continuing in their forward travel with constant magnitude until reaching the load. Also, as in the forward wave, both the reflected voltage and reflected current pass through zero simultaneously, (twice per cycle) and reach their maximum values one-quarter cycle later, because the line impedance Z_C is real.

So you now ask, are not the reflected voltage and current then also in phase with each other, like the forward voltage and current? *No!* As explained earlier, only the voltage *or* the current changes polarity during reflection, *but not both,* so they *cannot* be in phase. In fact, the maximum of the reflected voltage is positive when the maximum of the reflected current is negative, and vice versa, which means, as stated earlier, the reflected voltage and current are *always* 180° out of phase with each other. However, does the phase really matter here? *Indeed it does—there is no other relationship more important to wave propagation on a transmission line!* ***The 180° out-of-phase relationship between the reflected voltage and current is directly responsible for the development of the standing wave, and for the change in the input impedance of the transmission line from Z_C to a complex value when the load terminating the line is mismatched.***

To continue, although the forward and reflected waves travel separately in opposite directions, they are inescapably related to each other through their common line and load characteristics. Their respective voltages and currents add vectorially at every point along the line, forming the standing waves as the two waves slide past each other. In other words, the phasor sum of the forward and reflected voltages (or currents) at every point along the line produces the amplitude of their respective standing wave at each corresponding point on the line. In addition, the amplitude and angle of the resultant phasor voltage divided by the corresponding resultant phasor current determines the complex line impedance at each corresponding point on the line. Hence, the polarity relationships between the voltages and currents in the forward and reflected waves determine both the characteristics of the standing wave and the line impedance appearing at every point along the line, including the impedance appearing at the input terminals. We will explore the development of the standing wave in more detail later in this chapter, and again from a different viewpoint in Chapter 9.

Sec 3.2 Wave-Travel Analysis

Some people have difficulty in understanding the polarity relationships between voltage and current waves on transmission lines, so now let's examine polarity and the establishment of a polarity reference for convenience. Let us consider a single wave traveling on a two-conductor line. By following conventional current flow we can select the appropriate conductor as the voltage-polarity reference for a given direction of wave travel which causes the voltage and current maxima to occur with the same polarity. This polarity relationship may be reversed simply, either by selecting the opposite conductor for the voltage reference or by reversing the direction of wave travel. Obtaining the opposite polarity or phase relationship by reversing the conductors is a simple enough concept, but obtaining it by reversing the wave-travel direction has been a point of confusion for many people.

To reduce the confusion, a set of simple current-flow diagrams showing both ac and dc treatment is presented in Fig 3-1. Use of dc with zero-center meters as indicators make the explanation of polarity easy. Conventional needle movement is to the left for negative polarity and to the right for positive. Once polarity is clear, the battery may be replaced with an ac generator, and phase will also become clear using the waveforms as indicators. The circuits in A and B of Fig 3-1 are the same as in C and D, respectively, except that the voltmeter terminals have been reversed. Observe that the wave or energy-flow direction and voltage-polarity reference selected in A cause line voltage and current flow to be in the same polarity. Now notice that reversing either the wave direction (as in B) or the voltage reference (as in C) both result in *opposite* voltage and current polarities, as stated previously. Observe also that reversing both direction and voltmeter reference polarity (as in D) again results in line voltage and current flow with the same relative polarity, though reversed from A. It may be helpful at this point to perceive that changing the wave or energy-flow direction is equivalent to reversing the terminal connections of the current meter, because the source changes sides. This is the key to understanding the polarity-reversal problem, because for a given voltage-polarity reference, the current-flow direction must reverse when the wave-flow direction reverses.

Based on these conditions, assume a generator is now placed at each end of a single two-conductor transmission line. Then, a reference that is selected to make the voltage and current in phase on the line for one generator will result in 180° out-of-phase voltage and current for the other generator. This is the situation that exists with the mismatched RF transmission line — a source generator at one end of the line and the reflection generator at the other end. By selecting the conventional reference that makes forward voltage and current in phase with each other, it verifies that reflected voltage and current are 180° out of phase with each other *(Ref 35, p 23)*.

It is of interest at this point to be con-

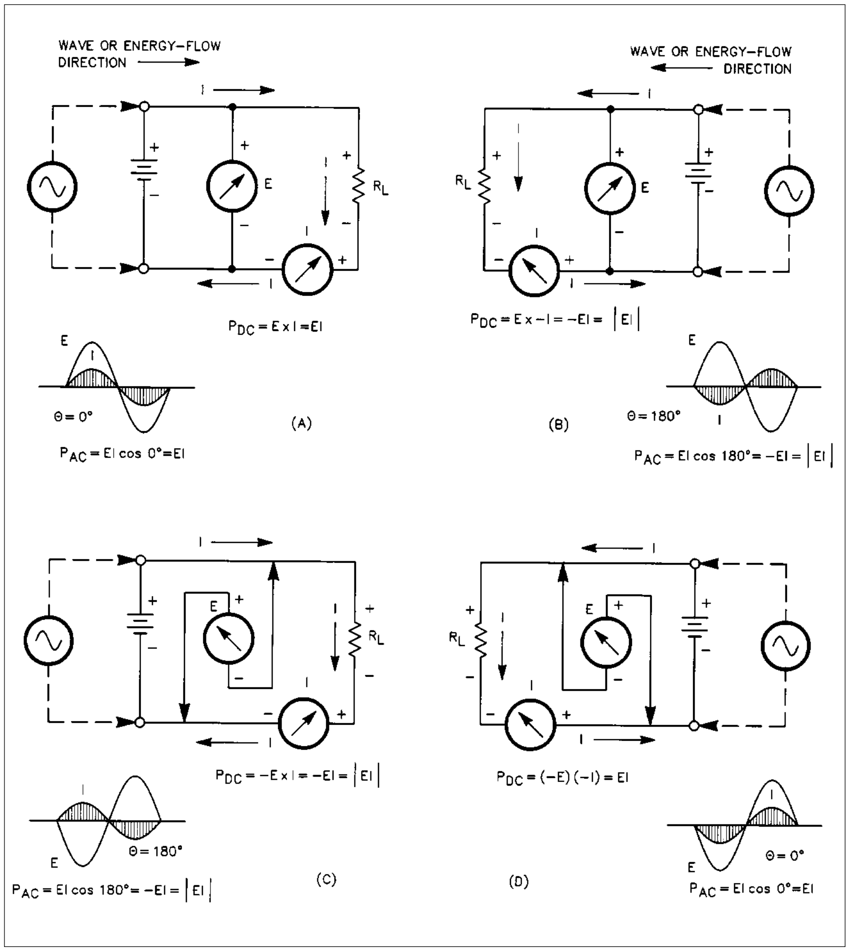

Fig 3-1—Illustrating relationships between current and various combinations of voltage reference and energy-flow direction.

cerned with the nature of the power in the forward and reflected waves. Some writers contend erroneously that the voltage-current phase relationship in the reflected wave is 90°. If this were true, then the reflected wave would contain only reactive volt-amperes, and no real power. The evidence presented above disproves this contention since we have seen that the voltage-current phase relationship in the reflected wave is 180°, not 90°. Moreover, we will certainly agree that if real power is conveyed in A of Fig 3-1, it is also real power in B, or C, even with reversed current-meter or voltmeter terminals. We will agree that *real* power P equals EI cosine θ, in which cosine θ is the power factor. It matters not whether the phase angle is 0° or 180°, because cos 0° = 1, and cos 180° = –1. This simply denotes the polarity difference discussed above.

When conductor spacing in a transmission line is restricted to the near field, i.e., a spacing that is a very small fraction of a wavelength, the fundamental principles governing transmission-line propagation are the same as those which govern all general ac circuit relationships, including electric power transmission. From these principles we know that real power flows for all values of θ in all four quadrants, except when θ is 90° or 270°, where cosine θ is zero, yielding zero power factor. However, whenever phase θ is other than 0°, 90°, 180°, or 270°, both real power and reactive volt-amperes are present. Nevertheless, at 0° or 180°, only real power exists because the absolute value of the power factor is 1.0 in either case. This clearly proves that reflected power and forward power in a transmission line are both *real* power, and that no fictitious power, or reactive volt-amperes, exists in either one. This is indeed true because, as I have shown earlier, the voltage and current in the forward wave are always in phase, and the voltage and current in the reflected wave are always 180°, out of phase.

The misunderstanding concerning *real*-versus-*reactive* power in the reflected wave arises in part from confusion between traveling and standing waves, because of insufficient familiarity with both types. To broaden the familiarity, I have concentrated first on the *traveling* waves. From the physical viewpoint, as stated earlier, standing waves are derived from the resultant interaction between the forward and reflected traveling waves. Thus, sufficient knowledge of traveling waves is essential before one can correctly understand the formation of standing waves and other correlated phenomena occurring on the transmission line. These other phenomena will become apparent as we proceed.

Now that you have a reasonably enlightened background concerning the ingredients of standing waves, we'll begin exploring the details of their development. Then it will be appropriate to return to the real-versus-reactive power misunderstanding for a few brief comments to clear away any remaining confusion.

Sec 3.3 Vector Graph Explanation and Standing-Wave Development

The newly launched reflected voltage and current waves, in their rearward travel toward the generator, add vectorially to their respective forward waves at every point on the line, with the result that their continuously changing relative phase differences along the line cause alternate cancellation and reinforcement of the voltage and current distribution on the line. This addition produces a continuous variation in the resultant voltage E and current I on the line that results in the formation of the standing wave and a change in the input terminal impedance, Z_{IN} , from the initial value of the line characteristic impedance, Z_C.

A physical picture of this complex relationship enhances the understanding of the phenomenon. Accordingly, the W2DU Vector Graph of Fig 3-2 graphically illustrates the progressive phase relationships between the forward and reflected voltages and currents as they travel in their respective directions along the line. (A larger version of Fig 3-2 appears on a foldout page near the back of this book.) Circular insets at every 22.5° (λ/16) along the line show accurately scaled phasor diagrams for visual comparison of the amplitude and angles of the forward and reflected waves relative to their position along the line. The reference for the angular distance along the line begins at the load termination point (the reflection plane where length len° = 0°, drawn at the right), with angular movement progressing clockwise toward the generator. These phasor diagrams superimposed circularly around the Smith Chart present certain symmetrical voltage-current phase-angle relationships with respect to line length, or distance from the load, that are not obvious in previously published displays *(Ref 2, p 72; Ref 17, p 146; Ref 18, p 110)*. In addition to providing a new way to show the formation of the standing wave, Fig 3-2 provides a new visual basis to explain many other aspects of transmission-line operation, such as the development of capacitive and inductive

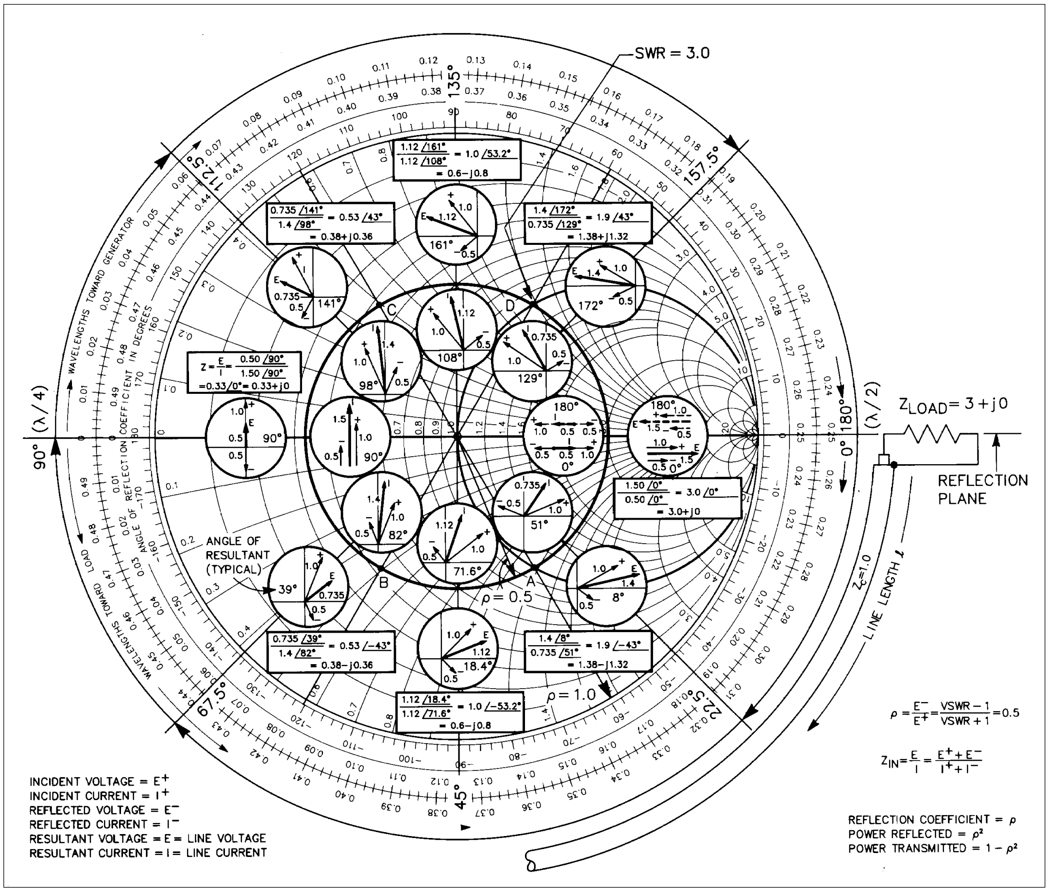

Fig 3-2—The W2DU Reflection Mechanics Vector Graph. For greater readability, this graph is printed in larger size on a foldout page near the back of this book.

components of complex impedance, the understanding of λ/4 impedance inverting action, the impedance-repeating phase-inverting action of the λ/2 line, and the reciprocal relationship between impedance and admittance.

For this illustration we are letting the line impedance Z_C = 1.0, which is the basis for handling normalized impedances on the Smith Chart, and we have terminated the line with a pure resistance of 3 ohms, three times the line impedance Z_C. Hence, a circle representing an SWR of 3.0 is shown. The line length len° is measured from 0° at the reflection plane, and is represented by clockwise rotation around the chart.

Phasors representing voltage are displayed in the insets outside the 3:1 SWR circle. Phasors marked "+" represent incident, or forward voltage (amplitude = 1.0), phasors marked "–" represent reflected voltage (amplitude = 0.5), and the phasors marked "E" represent the resultants of the forward and reflected phasors, both as to length and angle at the corresponding points on the line. (The resultant voltage, E, is also the *line voltage* appearing at the corresponding points. The corresponding current phasors appear in the insets directly opposite and inside the SWR circle. The resultant phasors marked "I" represent *line current*. The number in degrees shown with each phasor set indicates the angle of the resultant in that set. The phasor lengths are proportional to the amplitudes of their respective voltages and currents at each point along the line. By setting the lengths of the forward phasors equal to 1.0, the length of the reflected phasors inherently indicates the amplitude of the reflection coefficient, ρ. Hence, the lengths of the reflected voltage and current phasors in this example are 0.5 because ρ = 0.5 for an SWR of 3.0 from the relationship

$$\rho = \frac{SWR - 1}{SWR + 1} \qquad \text{(Eq 3-2)}$$

While the phasor lengths are proportional to each other, the lengths are not scaled to the chart dimensions. However, the various phasor diagrams on the graph contain the necessary amplitude and phase information to define both the standing wave and the impedance at each corresponding point on the transmission line.

The SWR = 3.0 (or ρ = 0.5) circle is based on the chart scales, and may be observed to have a radius of one-half the chart radius. The perimeter of the chart has a radius of 1.0, representing total reflection, that is, an infinite SWR. The radius of the ρ circle is thus directly proportional to the reflection coefficient, ρ. Thus, an SWR or ρ circle may be constructed for any value of reflection by making the radius equal to either SWR or ρ.

To make valid phase comparisons between various points on the line, wave motion has been frozen at an arbitrary point in time so that all phasors are shown in their true positions relative to each other. It makes no difference when the motion is stopped, but the symmetry of the presentation is enhanced by stopping the motion when the forward phasors at the reflection plane (where len° = 0°) are pointing in the zero direction in the standard polar coordinate system (to the right).

Observe that at any point on the line, the phasors representing forward voltage and current are always pointing in the same direction, indicating that they are in phase. In contrast, observe that the phasors representing reflected voltage and current are always pointing in opposite directions, indicating that they are always 180° out of phase, as explained previously. Also observe that at the reflection plane, len° = 0°, all components are in phase except the reflected current, which is 180° out of phase from all of the others. This is as it should be when the terminal impedance lies between Z_C and an open circuit. These phasor relationships graphically illustrate the concluding comments of Sec 3.2.

With clockwise travel from the reflection plane toward the generator, observe that each forward and each reflected phasor rotates the same number of degrees as those in the angular change in position along the line. But note the following statement carefully, because it is a crucial element in understanding the mechanics of wave travel on mismatched transmission lines: *The forward-wave phasors (+) rotate counterclockwise (phase leading), while the reflected-wave phasors (–) rotate clockwise (phase lagging).* For example, at 45° from the reflection plane, the forward-voltage phasor is at +45°, while the reflected-voltage phasor is at –45°, for a total phase difference of 90°. *Thus, for every degree of motion along the line, the relative phase angle between the forward and reflected voltage changes two degrees.* (The relative phase angle between the forward and reflected voltage is θ, which determines the *angle* of the reflection coefficient. Also observe on the graph that the degree scale labeled "Angle of Reflection Coefficient" increases at twice the rate of that of the line-length scale, len°.) This phenomenon becomes clear when you consider that in the distance from the reflection plane to our observation point at 45° along the line from the termination, the reflected wave has traveled twice as far as the forward wave. From the observation point, the forward wave travels only to the reflection plane, while the portion of the forward wave that is reflected travels an equal additional distance in returning to the observation point.

Now let's see what happens in traveling 90°, or λ/4 from the load toward the generator. Beginning at the load, where len° = 0°, we see that the forward and reflected phasor voltages (1.0 and 0.5 respectively) are exactly in phase with each other and adding, yielding the reinforcement mentioned earlier, indicated by the resultant E phasor length of 1.5. But at the point 90° toward the generator, the forward and reflected voltage phasors have each rotated 90° in opposite directions, so they are now 180° out of phase and canceling, or subtracting, with the resultant E phasor length of 0.5.

Continuing on to len° = 180°, or λ/2 from the load, we see that both voltage phasors have now rotated 180°, having continued rotating in opposite directions. Hence the forward and reflected phasors have rotated 360° relative to each other, and are again exactly *in phase with each other,* and reinforcing to 1.5 volts, *but note that all phasors are 180° out of phase with those at the starting point at len° = 0°.* Consequently, the dashed phasor lines appearing at len° = 0° (really len° = 180°) indicate the phasor angles that occur 180° from the load relative to those at the load, clearly illustrating the impedance-repeating, phase-inverting properties of half-wavelength λ/2 transmission lines.

However, at all points between len° = 0° and len° = 90°, the resultant voltage phasor, E, is seen to diminish gradually from the maximum of 1.5 at len° = 0°, to the minimum of 0.5 at len° = 90°, and then increase back to the 1.5 maximum at len° = 180°. In Fig 3-3, the values of these resultant phasor amplitudes at each 22.5° point along the line have been plotted on the more familiar rectangular coordinate graph. The smooth curves connecting the plotted amplitude values generate the familiar standing-wave patterns of voltage and current. The curves verify the relationship

$$SWR = \frac{1+\rho}{1-\rho} = 3.0 \qquad \text{(Eq 3-3)}$$

which is the algebraic expression for the addition and subtraction of the forward and reflected voltage phasors explained above. With ρ = 0.5, the E and I curves of Fig 3-3 show how adding ρ to the forward voltage of 1.0 at len° = 0°", and then subtracting it at len° = 90° generates the 3:1 SWR curves of voltage and current. Also bear in mind that the amplitudes at any given point of the SWR curves of voltage and current indicate the *line voltage* and *line current,* respectively, at the corresponding points on the transmission line.

Line lengths greater than λ/2 (180°) are accommodated simply by continuing on around the Vector Graph circle again (Fig 3-2), repeating the same values encountered 180° earlier, and thus establishing the periodicity of the standing wave. Only lossless line is being considered here; correction factors for attenuation, which change the SWR circle into an inward spiral, are presented later. The basis for the phase- or polarity-reversing characteristic of the half-wave, or 180°, line may be observed on the Vector Graph: Note again that the specific phase of the voltage phasors at the len° = 180° point on the line is 180° from their phase orientation at the reflection plane, at len° = 0°.

The significance of the constant 180° phase difference between the reflected voltage and current emerges if we now compare the phase and amplitude of the current phasors with that of the voltage phasors just discussed. We see that at the reflection plane len° = 0°, where the forward and reflected voltage waves are in phase and adding to create a voltage maximum, the corresponding currents are out of phase and opposing to create a current minimum. And at len° = 90°, where the voltage waves are out of phase and opposing, resulting in a minimum, the currents are *in phase*, resulting in a maximum. This is the reason, illustrated graphically, why *the maxima and minima of the voltage standing wave are always separated by 90° from the corresponding maxima and minima of the current standing wave.* This phenomenon is caused specifically by the constant 180° phase difference between the reflected voltage and current, as stated earlier and as shown here on the phasor display.

You can make a visual comparison of the voltage and current phasor resultants in Fig 3-2 (both angular positions and amplitudes) with the corresponding positions along the plot of Fig 3-3. This comparison should enhance your understanding of the important concept of a 180° voltage-current phase difference in the development of the standing wave. The concept is of further importance because, as we proceed, you'll see that this phase difference is the basis for the impedance-transforming properties of the transmission line, including the λ/4- and λ/2- sections (which are only two specific conditions of the general case).

As we discuss the concept of impedance in Chapter 4 and later solve matching problems, try to conjure up a mental image of the action occurring on the line. At times, this will be more helpful in understanding the difficult concepts than through logical reasoning alone. It is especially important to have a clear image of the formation of the reflected wave. Remember that the reflected wave must be considered as a *separate traveling wave.* It is identical to the forward wave except for the *direction* of travel and, usually, the magnitude. This point is important, because it helps us keep in mind that the reflected voltage and current waves travel the line 180° (not 90°) out of phase with each other, and thus transfer real voltage and current, and real power during their travel. It is essential to the process of reinforcement and cancellation of voltage and current along the line in the formation of the standing wave that *real power* be conveyed in the reflected wave, as if it had been developed by a separate source of power at the load end of the line. It will also become clear in the chapter to follow why the impedance along a line changes in the presence of reflections because ***real power*** *is flowing in both directions on the line (Ref 2, p 70; Ref 35, p 24; Ref 42)*.

I stress this point because, as mentioned previously, some writers have presented the erroneous viewpoint that the reflected wave conveys no real power, with the argument that the reflected voltage and current are 90° out of phase with each other and are therefore wattless. The reflected wave would indeed convey zero power if its voltage and current really were 90° out of phase, but the argument is incorrect because, as explained previously and shown in the Vector Graph, they actually travel 180° out of phase. At least three writers, Woods *(Refs 67 and 97)*, DeLaMatyr *(Ref 98)*, and Drumeller *(Ref 68)* would have us believe otherwise. It is easy to reach the wrong conclusion, however, because of the lack of a clear image of the reflection process and the somewhat complicated wave mechanics on the line. Woods and Drumeller have apparently confused the *reflected wave* with the *standing wave*, while all three writers have apparently confused *reflected* voltage E^- and *reflected* current I^- with *line*

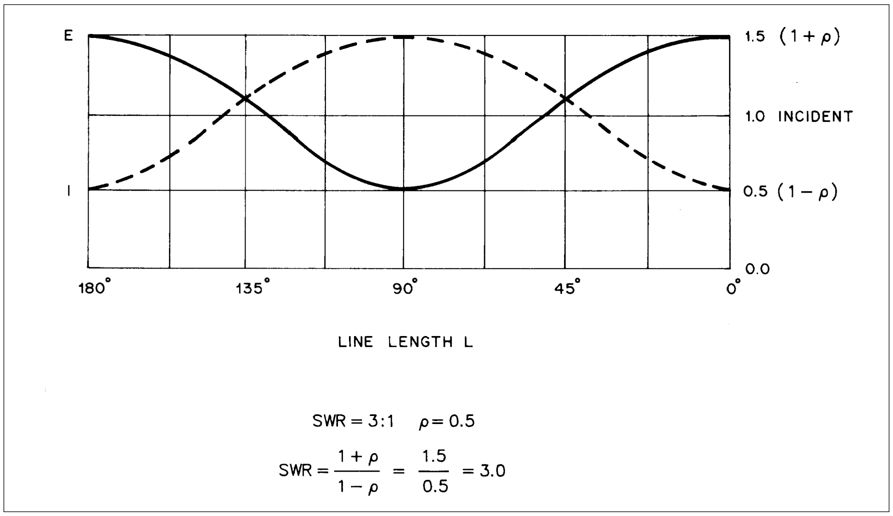

Fig 3-3—Illustrating how the voltage and current standing waves relate to the magnitude of the reflection coefficient, ρ.

voltage E and *line* current I. Drumeller says ". . . application of logic has demonstrated that reflected power is a vicious fiction." Unfortunately, Drumellers's logic is flawed, because, as explained in detail earlier, we know that reflected power is indeed *real, not fictional.* The true nature of the reflected wave as a separate electromagnetic traveling wave must be appreciated. It is also interesting to note that Woods states, ". . . re-reflection of power at the input end (of the line) is impossible to accept since the necessary conditions of impedance mismatch are not present." However, his Fig 2 shows a circuit which, when properly tuned to resonance as in all pi-network coupling circuitry, forms a conjugate impedance-matching network that *totally re-reflects all reflected power reaching it.* (There will be much more discussion in later chapters concerning the action of conjugate matching in relation to re-reflection of reflected power that reaches the pi-network.) This is also true in his Fig 1, since he implies that his final amplifier is loaded and tuned properly. Now the very essence of this impedance-matching network is its totally reflecting mismatch for reflected waves traveling toward the generator, while presenting a perfect match for forward waves traveling toward the load. Misunderstanding of this important basic concept is widespread among us Amateurs. This concept is basic to the operation of the pi-network for matching the output load impedance Z_L of the RF amplifier to its source impedance Z_S. The mechanism behind its operation involves wave interference and reflections, which are described in detail in Chapter 4, again using the Vector Graph as a visual aid, and again, from a different viewpoint, in Chapter 19, and from still another viewpoint in Chapter 23.

Mistaking the 180-degree difference in phase between the reflected voltage and current for the *90-degree difference of position on the line* that exists between the voltage and current maxima (or minima) of the standing wave (described a few paragraphs earlier and shown in Fig 3-3) is understandable. And the similarity between *line* voltage and *line* current behavior with the voltage-current relationship in conventional ac circuitry also makes it easy to understand why line voltage E and current I are being confused with reflected voltage E^- and current I^-. This is because, in addition to the in-phase line voltage and current components, which convey only the net power flow, line voltage and current do contain reactive components which are out of phase with each other when reflections exist on the line. Obviously, these reactive components convey no real power. Unfortunately, some people who are well versed in ac circuitry using lumped constants, but who are less familiar with the principles of transmission lines, sometimes make the error of assuming that the two circuit types are identical in electrical performance. Hence, we should be wary of making unwarranted comparisons that can bring about disastrous consequences of the type we are attempting to straighten out here.

Another erroneous argument set forth is that reflected power cannot be real power because it cannot perform work. I'll prove this argument false by showing how power in the reflected wave *does do work.* Now a simple RF voltmeter connected across the line indicates the line voltage at that point, and an RF ammeter in the line at the same point indicates the line current at that point. As stated earlier, line voltage and current are the *resultants* of the combined forward and reflected waves (E and I on the Vector Graph), the product of which, when multiplied by the cosine of the phase angle θ between them, yields only the net power that is absorbed by the load. Nevertheless, there are several devices in common everyday use, which selectively extract either the reflected or the forward wave from the line, independent of the standing wave. These devices permit separate measurement of the power associated with the waves traveling in either direction. One such device is the directional coupler. Another is the circulator. The circulator is a three-port directional device using port one as the input port. The wave reflected from a mismatched load that terminates port two is completely diverted away from the input feed line, and emerges only from port three. The reflected wave cannot get back onto the feed line to interact with the forward wave to develop a standing wave, or to change the line-input impedance at the source feeding port one. However, current flow through a matched resistor terminating port three develops I^2R heat equal to the amount of power reflected from the mismatched load that terminates port two.

A four-port hybrid coupler can be connected to perform in the same manner as the circulator. Directional RF devices most familiar to amateurs are the simple reflectometer SWR indicator and the directional wattmeter *(Ref 18, p 188, and Refs 38, 40, and 42)*. The meter in either instrument is actuated by RF power absorbed from one of the traveling waves on the line—either the forward wave or the reflected wave, as selected. If the reflected wave were wattless reactive power, no power would be available to actuate the meter movement in SWR indicators, or to produce heat from the current flow in the resistor on port three of the circulator. Furthermore, for power to become wattless on being reflected would violate the most general and fundamental of all physical laws, namely the law of conservation of energy *(Ref 35, p 25)*. Based on this law, if all the energy flowing in the line toward the load cannot be absorbed in or dissipated by the load, then that portion of the energy that is not absorbed must appear somewhere else. It cannot just disappear or cease to exist as if by magic. The reflected power recovered as heat in the resistance terminating the circulator is a typical proof of this fact, which is also proof *that reflected power is real power.*

Here is another way to express net power flow through a transmission line, one that enables us to break power down into its separate forward and reflected components. The final expression is obtained from the transmission-line power equations found in many engineering textbooks on transmission lines. The equations that follow are quoted from Johnson *(Ref 18)*.

$$\text{Power} = P = E \times I \qquad \text{(Eq 3-4)}$$

$$P = \frac{E^2}{Z} \qquad \text{(Eq 3-5)}$$

On RF lines,

$$P = E_{MAX} \text{ x } I_{MIN} \qquad \text{(Eq 3-6)}$$

And

$$P = \frac{E_{MAX} \text{ x } E_{MIN}}{Z_C} \qquad \text{(Eq 3-7)}$$

Now E_{MAX}, produced by $E^+ + E^-$ occurs as shown in Fig 3-2, where the forward and reflected voltages are in phase at len° = 0° and 180°, and E_{MIN}, produced by $E^+ - E^-$, where they are 180° out of phase, λ/4 away at len = 90°. Later we shall see that both the resultant voltage E and resultant current I are non-reactive at these points on the line, while being reactive everywhere else along the line between these points. However, because E_{MAX} and E_{MIN} are also non-reactive at these points on the line, respectively, the product of these voltages divided by the line impedance, Z_C, *yields the net power flow, exactly*, as expressed in (Eq 3-7). But recalling that E^+ and E^- are always non-reactive, we can replace the term E_{MAX} with $E^+ + E^-$, and the term E_{MIN} with $E^+ - E^-$ and thus

$$P = \frac{E_{MAX} \text{ x } E_{MIN}}{Z_C} = \frac{(|E^+| + |E^-|) \text{ x } (|E^+| - |E^-|)}{Z_C} \qquad \text{(Eq 3-8)}$$

Multiplying out the numerator terms gives the separate desired forward and reflected components of power.

$$P = \frac{|E^+|^2}{Z_C} - \frac{|E^-|^2}{Z_C} = \text{net power flow} \qquad \text{(Eq 3-9)}$$

The first term on the right of the equal sign in Eq 3-9 expresses the power associated with the forward wave and the second term expresses the reflected power. This simple separation of power into two components, each associated with one of the traveling waves, can be performed on a lossless or low-loss line, where the characteristic impedance Z_C is real. However, if the line has appreciable loss, the interaction of the two waves gives rise to a third component of power, which we need not consider, because lines normally used by amateurs are generally in the low-loss category *(Ref 18, p 150; Ref 37, p 129)*.

This separability of the forward and reflected powers forms the physical basis for the operation of reflectometers and directional wattmeters. These devices sense either the forward or reflected component by taking advantage of the 180-degree out-of-phase relationship of the reflected voltage and current, while the forward voltage and current are in phase with each other.

Bruene's published explanation of directional wattmeter operation is a classic paper *(Ref 38)*. In these instruments, a sample of the voltage across the line is added to a sample of voltage derived from the current in the line. When the voltage and current samples are adjusted to the correct amplitude relationship determined by line impedance Z_C, the difference between the two samples derived from the reflected wave cancel, while the sum of the samples represents only the voltage of the *forward* wave. But by reversing the polarity of either the voltage sample or the current sample, the difference between the two samples now derived from the forward wave cancel, while the sum of the samples represents only the voltage of the *reflected* wave. The scale of a meter connected to indicate these voltage sums can now be calibrated in *power*, because the voltage squared is proportional to power. The reversal of the polarity of the voltage or current samples is accomplished in SWR indicators by simply changing the position of the FORWARD-REFLECTED switch.

It is also important to note that the *meter-needle movement in simple SWR indicators is directly proportional to the voltage reflection coefficient ρ.* In other words, *the SWR indicator measures the reflection coefficien ρ, not SWR,* but the meter scale is graduated to convert reflection coefficient to SWR. The scale conversion is derived from the mathematical expression, presented earlier,

$$SWR = \frac{1 + \rho}{1 - \rho} \quad \text{(Eq 3-2)}$$

This scale calibration can be verified by first adjusting for a full-scale reading in the forward-power position, and then switching to the reflected-power position with a 3:1 SWR on the line. Because a 3:1 SWR results in the reflection coefficient (= 0.5, indicating that the voltage of the reflected wave is half that of the forward voltage. Under these conditions, as in our previous example, the needle of the SWR meter will point *to exactly half scale,* but the reading at the half-scale point on the SWR meter is "3". Check your own SWR indicator—I'm betting the "3" is at half scale on *your* meter!

The marvelous feature of the directional wattmeter is that, when properly calibrated, it indicates the *true power* in the transmission line regardless of the impedance of the load terminating the line; the load can be either an impedance match or a mismatch, and it can be either reactive or non-reactive. The meter achieves this because the forward power is equal to the sum of the line-input power and the reflected power. Hence, in the forward-power position, the meter indicates the total power that is incident on the load. In the reflected-power position the meter indicates the amount of the forward power which was *not absorbed* by the load, but which adds to the line-input power from the transmitter at the line input, or at whatever point in the line the match is performed *(Ref 18, p 191)*. Hence, the difference between the forward and reflected power readings is the *net power flow* in the line at whatever point the wattmeter is inserted. In a lossless line, the net power flow indicates the line-input power, which is the absorbed power exactly; the two are identical anywhere on the line. In a line with attenuation, the meter indicates the line-input power if it is placed at the line input, or it reads the absorbed power if it is placed immediately ahead of the load. The difference between these readings is related to the line attenuation. Of course, for practical reasons there may be small, or even large errors in the actual results from obtained by SWR measurements—for one example, diode non-linearity at various power levels *(Ref 40)*.

A further explanation of why the concepts discussed above are realized in practice, and the beginning of a discussion on the wave mechanics of impedance matching appear in Chapter 4.