Chapter 4

A View Into the Conjugate Mirror

(Adapted from *QST*, October 1973, edited June 1999)

Sec 4.1 Introduction

In Chapter 3 I discussed basic concepts concerning the generation of wave reflections on transmission lines, wave propagation along the line, and the development of standing waves. I then explained mathematically how RF power traveling on a transmission line is separated into its forward and reflected components. I concluded with mathematical expressions representing forward and reflected power as evidence, which proves that, if the line is mismatched at its termination, RF energy, or power, does indeed travel in both directions on transmission line. It then followed logically to explain how the power separation is physically realized by directional devices, such as SWR indicators and directional watt meters. While learning about wattmeter operation, and how to interpret the indications, we observed that, when the line is terminated in a mismatched load impedance and conjugately matched at the line input, the reflected power is re-reflected by the matching device. Due to the addition of the reflected power and the source power at the matching point, the incident or forward power in the line between the matching point at the line-input and the load is greater than the power supplied by the source generator. In this chapter I explore this transmission-line phenomenon in detail, because it is of great importance to the amateur in relation to the operational flexibility of the antenna system. Appreciation of the fundamentals involved in this seemingly anomalous situation will free you from the prevalent opinion that we are restricted to operating with little or no mismatch at the junction of the transmission line and the antenna terminals.

The explanation of directional wattmeter operation in Chapter 3 should clarify why the forward power appearing on the line between a transmatch and a mismatched load (the antenna) is greater than that delivered by the transmitter. This is a normal condition that enables a mismatched load to absorb all the power delivered by the transmitter, while at the same time reflecting a portion of the total power it receives. To do this, the load must receive enough more forward power than what is delivered by the transmitter so it can absorb the power delivered and still reflect the portion demanded by the mismatch. The basis for understanding this rather subtle concept lies in the wave mechanics behind the principles of impedance matching introduced in Chapter 1 and defined in Chapter 2. As far as I know, the *wave* aspect of this subject has been presented in the literature only by Slater, Alford, *(Refs 35, 39,* and this author in Chapter 23.)[1] Perhaps this restricted exposure may account for some of the confusion in this area among engineers and amateurs alike. For example, the many "cook book" recipes and graphic directions for stub-matching a mismatched line tell "how" to do it, but offer little insight concerning the wave mechanics through which the match is accomplished. (but this author does in Chapter 23.) However, this insight goes to the heart of the transmitter-to-line coupling problem. It clarifies how the reflected wave becomes re-reflected at the matching point. If the matching did not produce this effect, the reflected wave would travel along the line back to the generator, reducing the amount of power made available by the generator due to the mismatch now appearing to the generator at the input of the line. *(Ref 19, p 37).*

Sec 4.2 The Normalizing of Impedances

I will shortly be discussing the reflection mechanics involved in impedance matching, using the technique of employing a matching stub on a mismatched transmission line. In this discussion we use *normalized* impedances, represented graphically on the Smith Chart W2DU Vector Graph, Fig 3-2. So I'll now digress briefly to explain normalization. We are rather accustomed to thinking of 50 ohms as a standard *system* impedance because of the preponderance of RF components and coaxial lines using that impedance. However, many calculations are greatly simplified by using *normalized* impedances, in which all impedance values have been divided by the system impedance. The characteristic impedance Z_C of the transmission line being used generally determines the system impedance. Normalizing an impedance by dividing it by Z_C amounts to a change of scale such that the unit of impedance is 1 chart ohm, rather than Z_C ohms. The W2DU Vector Graph uses the normalized system to take advantage of the simplification in the calculations. To obtain normalized values occurring in a system based on any specific impedance, simply divide all impedances by the system impedance. For example, in a 50-ohm system, 150 ohms becomes 3.0 chart ohms. To convert back to the 50-ohm system, simply reverse the process and multiply the normalized values by 50. For example, the normalized impedance $z = 0.6 - j0.8$, found at len° = 45° becomes $Z = 30 - j40$ when de-normalized. When working with the Smith Chart the normalized values of resistance and reactance appearing on the Chart are often called *'chart ohms'*. Also note that normalized values of impedance components are customarily shown in lower case, and non-normalized values are capitalized.

Sec 4.3 Reflection Mechanics of Stub Matching

We will now examine the mechanics of wave interference and reflection, which is the basis for all impedance matching operations. An approach that will provide a clear understanding of the wave action that occurs in the matching process uses complex reflection coefficients to represent the magnitude and phase of the waves at various points on the line. The complex reflection coefficient, represented by $\overline{\rho} = \rho\angle\theta$, with magnitude ρ and angle θ, were introduced in Chapter 3, and are discussed in greater detail in Chapter 9. As stated along with the definition of the conjugate match in Chapter 2, the matching is accomplished by inserting a non-dissipative mismatch at the matching

1 The virtual one-way circuit will be either an open circuit or a short circuit, depending on the value of the resistive component R in the terminating load in relation to the characteristic impedance Z_C of the transmission line. If the load resistance R is greater than ZO the one-way circuit will be a virtual open circuit, conversely, if R is less than Z_O it will be a virtual short circuit.

point, which produces a complementary reflection that compensates and cancels the waves reflected from the mismatched load. It was also stated that matching can be achieved by a correct adjustment of either the final tank tuning circuit *(Ref 4, Part III)*, or of a line-matching network (antenna tuner or transmatch) if one is used. Because stub matching uses the same principles as used with a matching network, and is easier to visualize, we will use the stub technique to demonstrate the wave mechanics that perform the matching function.

In stub matching, the stub provides what may seem like an anomaly—a non-dissipative discontinuity, or mismatch. While we usually think of the stub as providing a *match* (which it does), rather than a mismatch, we will discover that the impedance match achieved by the stub results from constructive and destructive wave interference introduced by the stub. The destructive interference causes mutual cancellation of two complementary reflected waves generated by two complementary mismatches. One of the two waves is the original undesired wave reflected from the terminating load mismatch, the wave that needs to be eliminated to achieve the match. The second wave is a new reflected wave generated by the stub mismatch, introduced to cancel the undesired wave. This new reflected wave generated by the stub is equal to the undesired load-reflected wave in both magnitude and phase *but with opposite phase sign.* The *destructive* wave interference between these two complementary waves at the stub point causes a complete cancellation of energy flow in the direction toward the *generator.* This cancellation results from the *difference* between these two equal but oppositely phased reflected waves. Conversely, the *constructive* wave interference produces an energy *maximum* in the direction toward the *load,* resulting from the *sum* of the two reflected waves and the source wave. The cancellation of energy flow rearward from the stub, toward the generator, results in no reflected power on the line between the stub and the generator. This means that the impedance now appearing at the input of the line is simply the characteristic impedance Z_C of the line, and the SWR on the line between the stub and the load is 1:1, although the original SWR between the stub and the load remains.

The effect of the energy-canceling wave interference creates either a virtual one-way open circuit or a one-way short circuit at the stub point (the matching point) to waves traveling rearward toward the generator.[1]

How the wave interference creates the virtual one-way open or short circuit will be explained shortly. This one-way circuit blocks both the load-reflected waves and the stub-reflected waves at the stub point from any further rearward travel, and causes these waves to be totally re-reflected toward the load, in phase with the forward waves. Hence, the power in the re-reflected waves adds directly to the forward waves, causing the forward power to be greater than that delivered by the transmitter, as stated earlier.

This wave-interference mechanism which accomplishes the virtual one-way open or short circuit that is vital to the impedance-matching phenomenon will become more clear as we investigate the complex reflection coefficients of the load and stub mismatches with the aid of an example using the Vector Graph in Fig 3-2. In this example we are using *normalized* values of impedance Z, resistance R and reactance X. Here we observe a terminating load of $3 + j0$ chart ohms, which yields a reflection-coefficient magnitude $\rho = 0.5$, resulting in a 3:1 standing-wave ratio along the whole line. The SWR of 3 is shown by the dark concentric circle, which is the locus of impedances anywhere on the line when the terminating load is 3 + *j*0. This circle intersects the unity resistance circle marked 1.0 at the two points A and D, as shown. At these corresponding points on the transmission line, the resistive components[5] of the line impedance equals 1.0 times the characteristic impedance Z_C of the line. Hence, points A and D are appropriate positions for attaching a *series* matching stub to cancel the line reactance appearing at that point[2]. Points A and D on the graph are also intersected by reactance circles marked 1.15. At point A the reactance is negative, – 1.15 times Z_C, and at point D the reactance is positive, +1.15 times Z_C. Thus, the line reactance is – *j*1.15 ohms at stub point A, and + *j*1.15 ohms at stub point B. It is therefore evident that the line impedances appearing at points A and D are 1 – *j*1.15 and 1 + *j*1.15 chart ohms, respectively. It should be understood that when we relate these values to real transmission lines, whose characteristic impedance Z_O value is, for example, the commonly used 50 ohms, the line impedances above would be denormalized by multiplying by 50. Thus the real values of these line impedances would be 50 – *j*57.5 and 50 + *j*57.5 ohms, respectively.

The conditions for obtaining reflected-wave cancellation by wave interference are the well-known stub-matching requirements, as follows:

1) A series stub is placed where the resistance component of the line impedance equals Z_C, such as at points A or D on the unity-resistance circle, and

2) The stub is tailored to produce a reactance equal in magnitude and opposite in sign to the line reactance appearing at the stub point, which is the reactance that results from the phase relationship between the forward and reflected waves superposed at the stub point. Thus the stub and line reactances cancel to zero at the stub point, which is also the matching point. *(See Ref 2, p 116 and Ref 19, p 97).*

This sounds almost like the conjugate-match definition itself, doesn't it? The correct point for inserting a reactance-canceling stub in series with the line nearest the load is at point A on the SWR circle.[3] Alternatively, the stub may also be placed at point D, or at any λ/2 interval along the line from points A or D. This is possible because the line impedances are repeated at every λ/2 distance around the diagram, and along the line. However, it should be understood that a stub placed at point A must have a positive (inductive) normalized reactance, + *j*1.15 ohms, to cancel the equal, but negative reactance component of the line impedance at that point. Conversely, a stub placed at point D, must have a negative (capacitive) normalized reactance, – *j*1.15 ohms, to cancel the positive reactance component of the line impedance appearing there.

Using the complex voltage and current coefficients of reflection, magnitude ρ and angle θ, we'll now examine how reflections add at a stub matching point to produce the matching effect. We'll see why a directional wattmeter gives a true reading of forward power between the matching point and the load-power that is

2 For this example we are using series stubs to permit impedance treatment for the reader who is unfamiliar with admittance treatment. Both series and parallel stubs are used in commercial short-wave transmitting systems, where balanced open-wire transmission lines prevail. However, series stubs are not practical for use with coaxial transmission lines. Thus parallel, or shunt stubs are used more prevalently in general practice, but for some, using the shunt form would also require an explanation of admittance treatment, which is beyond the scope of this example

greater than that delivered by the transmitter when the line is terminated with a mismatched load. A little later we'll also see how these principles apply to practical feed-line matching networks, including the ubiquitous antenna tuner performing the matching at the input of the line.

To clarify the following operations used in explaining the matching function using reflection coefficients, it should first be understood that any impedance Z = R + *j*X may also defined in terms of its equivalent complex reflection coefficient $\overline{\rho} = \rho\angle\theta$. At point A, which is 30° from the load (at len° = 30° in Fig 3-2), while the normalized line impedance E/I is $1 - j1.15$ ohms, the unmatched voltage reflection coefficient is seen to be $\overline{\rho}_E = 0.5\ \angle{-60°}$. This means that the magnitude of the reflected voltage wave is one-half the value of the forward wave, and the phase of the reflected voltage wave lags the forward wave by 60° at point A. (The superposition of the forward wave and the lagging reflected wave at point A results in the formation of the normalized line reactance, $-j1.15$ ohms at point A.) A match can be effected by connecting an inductive reactance, such as a stub (or a lumped inductance) of + j1.15 chart ohms in series with the line at point A. The reactance-cancellation effect of the positive-reactance stub on the equally negative reactance of the line impedance is generally understood, but several points are not always clear regarding the effect on the component wave. What characteristics of the stub cause it to counteract the reflections from the load? Also, why does the stub cause the forward power to increase between the matching point and load?

In answer to these questions, let's first determine the reflection coefficient produced by the stub if it were inserted at point A in a perfectly matched line, i.e., with the line terminated with a normalized one-ohm resistance instead of the 3-ohm resistance appearing in Fig 3-2. In this condition we may analyze the reflection generated by the stub in the absence of any other reflection on the line. If we observe from a position just on the load side of stub point A with the stub attached, and look into the line toward the matched termination, we will see a pure resistance R equal to the characteristic impedance of the line, Z_C.

We know from transmission-line theory that, if we replace the matched line portion extending from the stub to the load with the terminating resistance R = Z_C alone, in series with the stub across the otherwise open-ended line, we may look into the line toward the stub from the generator and see the same conditions of reflection as were present before the line section was removed and replaced with the resistor. Thus, the series circuit comprising either the stub and the matching resistor, or the stub and line section terminated with the matching resistor alone, develops a $1 + j1.15$–ohm impedance on the line at the stub point. Consequently a reflection is generated by the stub at the stub point that is of precisely the same magnitude as that generated by the 3-ohm mismatched load in Fig 3-2, and precisely this same reflection is produced no matter where the stub is inserted in a matched line. The Vector Graph shows this impedance of $1 + j1.15$ ohms to appear at point D, for which the complex voltage reflection coefficient is $\overline{\rho}_E = 0.5\ \angle{+60°}$. Note that this is the same magnitude and phase *but of opposite phase sign* of the reflection coefficient appearing at point A resulting from the load mismatch of $3 + j0$.

Thus, the stub mismatch produces the same magnitude of reflection (and thus the same SWR) as was produced by

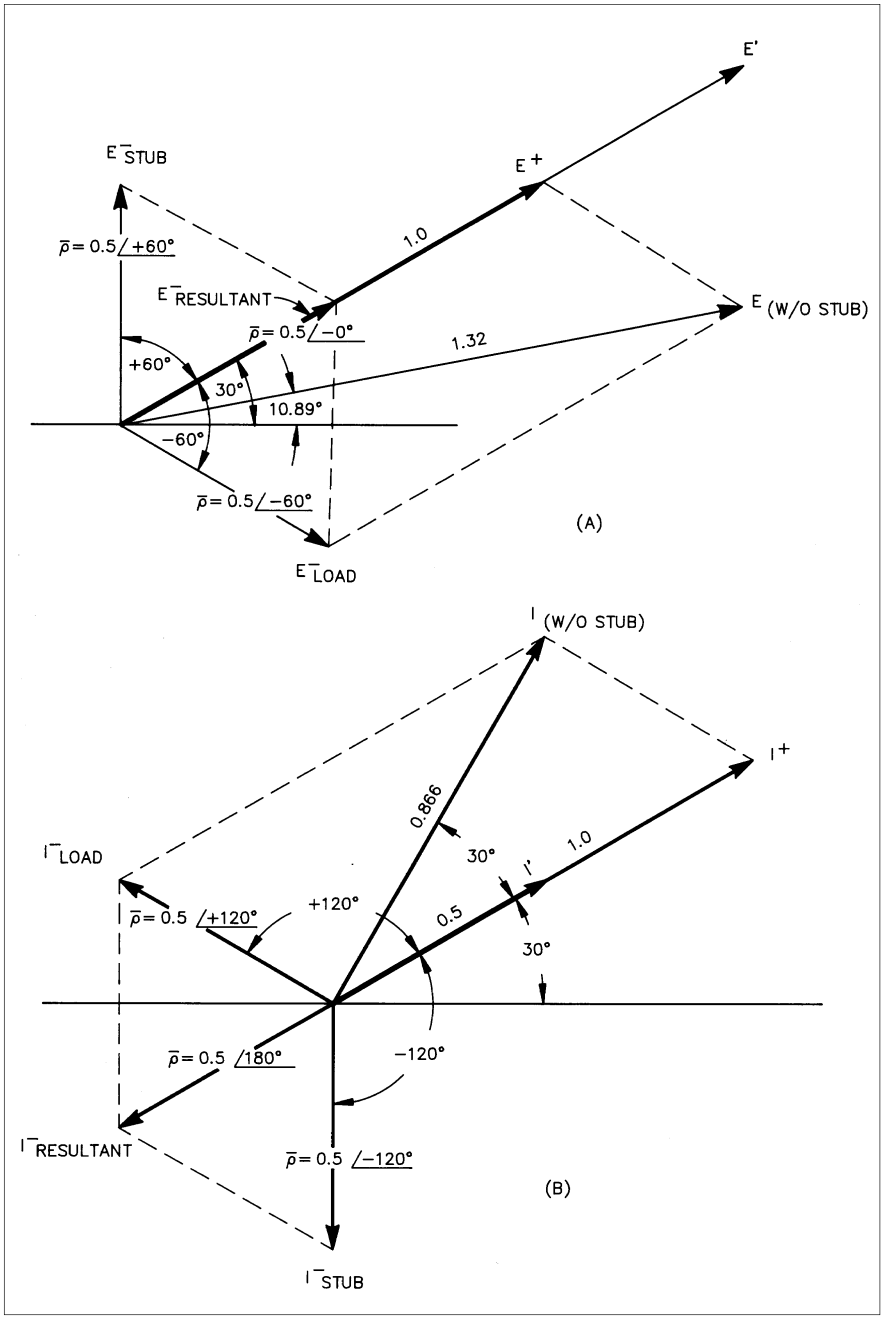

Fig 4-1 — At A, various voltage-vector relationships, and at B, various current-vector relationships at len° = 30°.

the load mismatch, but the stub-reflected voltage wave *leads* the forward wave by 60°, while the load-mismatch wave *lags* by 60°. If the series stub is now inserted at matching point A (corresponding to len° = 30° on the Vector Graph Fig 3-2) with the 3 + *j*0 load terminating the line, both the stub-mismatch and the load-mismatch reflections will appear simultaneously at the same point. As a result of their opposite-sign phase relationship, the *leading* stub-reflected wave and the *lagging* load-reflected wave cancel each other at the match point due to destructive wave interference.

The voltage reflection coefficients of the load (∠–60°) and stub (∠+60°) thus add vectorially to a resultant of zero degrees, $\overline{\rho}E$ = 0.5 ∠0°, which tells us that the resultant of the two voltage reflections is exactly in phase with the forward voltage wave at match point A, as shown in Fig 4-1A. The amplitude resulting from this trigonometric addition is considered later, but knowledge of these angular relationships should clarify the understanding of the mechanics of line-reactance cancellation by the stub.

Now that we know what is happening with the voltage waves, we also need to investigate the current waves to learn about the complex impedance Z = E/I appearing at the matching point. As defined earlier, reflected current is always 180° out of phase with reflected voltage. Hence, the complex reflection coefficient for current at point A on the line is found on the Vector Graph 180° away at point C, diametrically opposite the corresponding voltage-coefficient point. Thus, we find the current reflection coefficient for the load mismatch at point C with $\overline{\rho}_I = 0.5 \angle{+120°}$. Similarly, the complex reflection coefficient for current of the stub mismatch is found diametrically opposite point D, at point B, and is $\overline{\rho}_I = 0.5 \angle{-120°}$. Note that, similar to the case with voltage, the current coefficients of the load and stub are also of equal magnitude and phase, but of opposite phase sign. But while the voltage angles added vectorially to the resultant 0°, the current angles add to the resultant 180°. Hence, the resultant of the two reflected *current* waves is 180° out of phase with the forward current, as shown in Fig 4-1B. So we now have forward and reflected voltages in phase (at 0°), and forward and reflected currents 180° out of phase; hence, the wave arriving at the match point from the generator sees a perfect match. Why a perfect match to waves arriving from the generator? Because these specific magnitudes and angles of the reflected waves of both voltage and current send a significant message which we will now reveal. With reflected voltage and current phasors of equal magnitude, and the voltage phasor at 0° and the current phasor at 180°, a virtual open circuit to rearward traveling waves is established at the stub point, which I will now explain.

In the opening paragraphs of Chapter 3, I described the wave mechanics involved in a line terminated by an open circuit. There we learned that angle theta (θ) of the voltage reflection coefficient at an open circuit is 0° and that of the current is 180°. With an open-circuit termination, the voltage wave which is incident on the termination is totally reflected with no change in phase, while the current wave is totally reflected with a change in phase of 180°, a complete *reversal* of current polarity.

The significance of the reflection conditions appearing at the open circuit described above is that the phase relationships of the resultant reflected waves at the stub matching point which we established earlier θE = 0°) and θI = 180°), are identical to those which prevail in a line terminated in a real open circuit, so far as the reflected waves are concerned (but not the forward waves). Therefore, the effect of the two oppositely-phased reflected waves arriving simultaneously to superpose at the match point is to establish a virtual open circuit at the match point to the waves generated by the load and stub mismatches. Consequently, the waves from the two mismatches become totally re-reflected at the match point and undergo *open-circuit* phase changes, as described above. Hence, on re-reflection at the match point, the angle of the resultant re-reflected voltage wave, already at 0°, does not change phase. However, the resultant *current wave changes phase* by 180° on re-reflection. And because it was 180° out of phase with the forward current wave just prior to re-reflection, its

180° reversal on re-reflections places it also in phase with the forward current wave. Now that both voltage and current re-reflected waves are in phase with their corresponding forward waves, direct addition of the voltages and currents of the forward and reflected waves occurs at the stub-matching reflection point. Thus the conclusion: 1) On re-reflection *the power contained in the reflected waves adds directly to the forward power*, increasing it between the matching point and the load in the manner stated earlier. 2) *A 1:1 impedance match to source waves from the generator appears at stub matching point.*

Now let us consider that the above conclusion assumes a virtual open-circuit was established at the match point for the reflected waves traveling toward the generator. The assumption was based on the identical relationship between the reflection coefficients, which we established at the match point by wave interference and those known to exist at a real open-circuit termination. We can verify this assumption by an alternative method. The method is based on the discussion in Chapter 3 concerning the electric- and magnetic-field relationships that occur at an open-circuit termination.

Let us first observe the net value of all currents flowing at the match point at the instant the two superposed reflected currents form their resultant angle θ = 180°. At this instant we will see an initial sudden drop in resultant line current, I, because of wave cancellation as the reflected-wave resultant becomes aligned exactly 180° out of phase with the forward current. This drop is shown graphically in Fig 4-1B, where the original resultant current, I (as with no matching stub present) suddenly drops to the new instantaneous resultant value I' from the effect of the stub discontinuity.

Now recall the open-circuit field theory presented in the early part of Chapter 3. We know that when the current drops, the magnetic field also drops. The changing magnetic field produces an electric field equal to the energy reduction in the magnetic field. The new electric field then adds in phase to the existing electric field, producing a voltage increase at the match point. This voltage increase now starts a wave traveling in the opposite direction, which is now in the same direction as the forward wave, and thus adds to it. The increased electric field (now an enlarged *forward* electric field), as it moves toward the load, produces a new magnetic field. The new field is equal in magnitude but of opposite polarity to that of the original field. The new magnetic field now causes current to build up again to the same magnitude as the original reflected current, but of opposite polarity and direction. Thus, the new current wave is now also traveling in the same direction, and with the same polarity as the forward current wave. The new current wave adds to the forward wave, enlarging it just as the re-reflected voltage wave added to and enlarged the forward voltage wave.

By following these field-current-voltage reactions through their natural sequence of events, you can see that we have obtained the same conclusions as those previously obtained on the basis of reflection coefficients. This justifies the assumption that the resultant reflection coefficients at the stub-matching point have defined a virtual open circuit to the reflected waves, thus preventing any further travel of the reflected waves toward the source. The existence of the reflectance (ratio of reflected power to forward power) at the matching point is therefore verified, with the result that both the reflected voltage and current have indeed been re-reflected, and the power associated with them has thus been effectively added to the power contained in the source wave. Thus, when the line is terminated in a mismatch causing reflected power to exist on the line, the *sum* of the source and re-reflected powers (traveling only toward the load) must be greater than the power delivered by the generator alone, when conjugate impedance matching is performed at the input of the line. And since we have shown how the stub acts to counteract reflections from the load, our original questions concerning the stub characteristics have been answered.

Sec 4.4 Effect of Line Attenuation on Reflected Power

Now we'll consider the effects of line attenuation, alpha (α) with respect to the increase in the loss of power delivered to a load when the load is mismatched to the transmission line impedance, Z_C, but conjugately matched at the input of the line with a matching device such as an antenna tuner. When determining the effects due to attenuation on transmission lines, it is customary to first determine the pertinent parameters while considering the line to be lossless (zero attenuation), and then re-calculate to include the attenuation. The next step then is to determine the total forward power in the lossless line resulting from the addition of the power re-reflected at the match point to that from the source, the transmitter. Let's consider a mismatch that yields a reflection coefficient $\rho = 0.5$, resulting in a 3:1 SWR, and that the transmitter delivers 100 watts into the line. Because of the addition of the re-reflected power to the power delivered by the transmitter, after a few cycles of round trip travel to establish the steady-state condition, the forward power in the line with the 3:1 mismatch is 133.33 watts, and the reflected power is 33.33 watts. The 133.33 watts of forward power results from multiplying the 100 watts from the transmitter by the 'forward-power-increase factor' $1/(1 - \rho^2)$. With the reflection coefficient $\rho = 0.5$, ρ^2 is the *power* reflection coefficient, 0.25, and $(1 - \rho^2)$ is the power *transmission* coefficient, 0.75. Thus $1/(1 - 0.25) = 1/0.75 = 1.3333$, and 1.3333 times 100 watts equals 133.33 watts, which is incident power at the load. On the lossless line used for the reference, 100 watts will be absorbed in the load, and 33.33 watts will be reflected, because reflected power $\rho^2 = 0.25$, or 25% of the forward power. This leaves 75% absorbed by the load, and because 75% of 133.33 watts is exactly 100 watts absorbed in the load.

Now we'll apply the data obtained with lossless line to a realistic line with attenuation. As an example, the matched-line attenuation in 175 feet of RG-213 at 4 MHz is 0.5 dB, and our problem is to find the additional loss sustained with this line attenuation when the load mismatch is 3:1. If the load were perfectly matched to the line, for a 1.0 SWR, the 100 watts delivered by the transmitter would be attenuated to 89.13 watts by the 0.5-dB line loss during travel to the load. However, with a 3:1 mismatched load the *additional one-way* line attenuation *because of the SWR* is 0.288 dB.[4] We can now determine the power absorbed by the mismatched load by adding the 0.288 dB additional attenuation to the 0.5 dB matched attenuation, for a total of 0.788 dB. Then we reduce the 133.33 watts of forward power obtained above with *lossless* line by the 0.788 dB. The result is 83.41 watts absorbed in the 3:1 mismatched load. To continue, the forward power at the conjugate-match point at the line input is 124.78 watts (0.288 dB below 133.33 watts), and 111.21 watts (0.5 dB below 124.78 watts) reach the load. Of the 111.21 watts reaching the load, 27.80 watts (25%) are reflected, leaving 83.41 watts to be absorbed in the load. Of the 27.80 watts reflected, 24.78 watts arrive back at the input to join the 100 watts of source power to develop the 124.78 watts of forward power. The difference between the 89.13 watts absorbed in the *matched* load, and the 83.41 watts absorbed in the 3:1 *mismatched* load is only 5.72 watts. This small amount of power loss is insignificant, because when considering that an S unit normally refers to a change of 6 dB, the loss in this case amounts to less than 1/12 of an S unit. A circuit model illustrating the effects of the line attenuation in this example appears in Appendix 6, Fig 23-4B, and Example 5. Procedures for calculating these values appear Appendices 7, and 8. These values are typical of data obtained during actual routine measurements in a professional laboratory. They provide additional evidence that reflected power is real, not fictitious. If it were fictitious power, no more than 66.85 watts (75% of 89.13 watts) would be available for delivery to the 3:1 mismatched load. But the 83.41 watts actually absorbed in the mismatched load is 93.58% of the amount absorbed in the matched load, the additional loss of 6.42% being completely accounted for in the line attenuation encountered by the reflected power.

Sec 4.5 Matching Networks and Reflection Mechanics

Now we'll delve further into the wave-interference principles demonstrated with the stub technique in Sec 4.3. These principles also apply to both the resonant λ/4 series matching-transformer operation and the typical amateur "antenna tuner," line-matching network, or transmatch. In order to visualize the inherent generality of these principles, we need to develop some additional concepts concerning stub matching and embark on a somewhat different line of reasoning. As you can surmise from the example presented earlier, the fundamental principle behind the elimination of reflections is this: Have the original reflected wave canceled at the point where the elimination of the reflection is desired, by destructive wave interference from another wave of equal magnitude and phase but of opposite phase sign *(Ref 35, p 58, and Ref 39)*. A transmission line of the appropriate length, which has one end effectively open circuited and the other end short circuited, possesses the reflection-producing characteristics required to develop canceling waves of the correct phase in relation to the wave to be canceled.

Using other line arrangements can also develop canceling waves, but for the purpose of demonstrating the principles, I will use the arrangement just stated. Fig 4-2 shows how a stub of this arrangement performs the matching function in practice. Fig 4-2A is the conventional representation of a typical series-stub circuit (using the values of our previous SWR = 3 example), in which section F is called the *feed line*, section S is the *stub,* and section T is an impedance-transforming section which I will call the *transformer.* We'll now examine these in greater detail. To clarify the approach, Fig 4-2A is redrawn in Fig 4-2B, with the stub and transformer shown as one continuous straight-line section. This straight-line section will presently come to life as the heart of the wave-interference-producing mechanism found in all stub-matching operations. This is because its physical length will be adjusted specifically so that waves reflected at each end will return to the feed point with equal magnitude and phase, but with opposite phase sign.

We observed earlier that a voltage

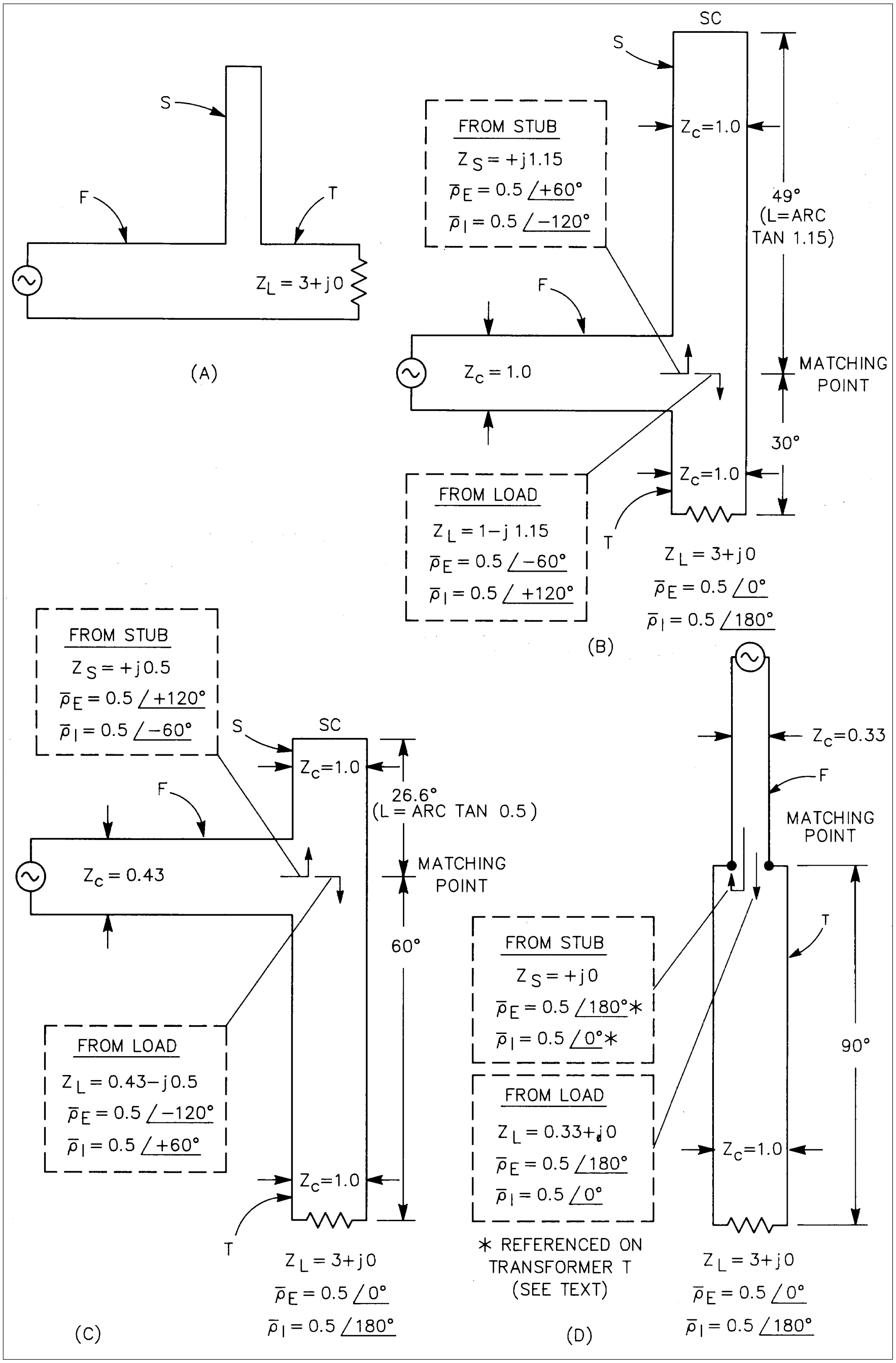

Fig 4-2 — At A, the concept of stub matching with a series-connected stub. At B, the same matching arrangement redrawn. At C, a similar matching arrangement, but with different stub and transformer line lengths. At D, a λ/4 line-transformer section evolves as the stub length goes to zero. SC = short circuit.

wave is reflected with zero phase change at an open circuit, or with any resistive termination *greater* than the Z_C of the line. And it is reflected with 180° of phase change from a short circuit, or from any resistive termination *less* than Z_C. With a current wave, the opposite is true. So from the viewpoint of phase angle in reflection behavior, with the 3 + *j*0-chart ohm load terminating the transformer, the load end of the straight-line section is considered as being open circuited and the opposite end short circuited. As can be seen, which end is open and which end is short circuited depends on the character of the load, i.e., whether the resistance in the load is greater or less than Z_C. In our example in Fig 4-2B, the load end behaves as an open circuit as far as wave reflection is concerned because R is greater than Z_C, while the opposite end (the stub) is short circuited. The action occurring in this line section in the process of developing the interfering wave-canceling relationship is as follows. Either a voltage or current wave is assumed to enter the line section at the feed-line entry point. The energy in the wave divides, one portion of the wave traveling toward one end of the section, and the other wave portion traveling toward the opposite end. After each wave portion encounters one reflection, the returning waves will each have the same absolute value of phase but opposite sign, or polarity, on return to the point of entry. This is true for both voltage and current.

The opposite phase or polarity between the two reflected waves arriving from opposite directions results because reflection at one end is accompanied by a 180° phase reversal, while reflection from the other end is not. As stated above, the phase reversal of one wave but not the other is caused by the opposite conditions of reflection at the two ends of the line, one end open-circuited and the other end short-circuited. Note in the dashed-line boxes in Fig 4-2B that in each case, the phase of the reflected wave (of both voltage and current) is of opposite polarity on opposite sides of the feed line as they return from the stub and load directions. The wave-entry point, where the feed line is attached, is the matching point. This point divides the line section into its two complementary portions: the *stub* portion, S, and the *impedance-transformer* portion, T. Electrically, each portion is the complement of the other, because the waves reflected from the end of each portion returning to the match point are complementary in their phase relationship and equal in magnitude. Herein lies the basis for the term *complementary mismatches* as used earlier, because each portion presents a complementary mismatch to the feed line.

Later I show that this complementary mismatch concept is of great importance to matching in general, because the complementary relationship holds no matter where the feed-line entry point is positioned on the stub-transformer line section. The importance prevails because the canceling wave and the reflected wave to be canceled will be of the same magnitude and phase, but opposite phase sign, at whatever point on the matching line section the feed line is attached. This is true with two provisions: (1) the characteristic impedance Z_C of the feed line section F must be the same as the resistive component of the transformed line impedance appearing on the transformer line at the feed point. And (2) the length of the stub portion S must be adjusted to produce a reactance equal and opposite in polarity to the line reactance appearing at the feed point in the transformer section. That length may be found from the expression:

$$S_L = \arc \tan \frac{jX}{Z_C} \qquad \text{(Eq 4-1)}$$

where

S_L = stub length in electrical degrees

jX = line reactance (obtained from the Vector Graph Fig 3-2)

Z_C = characteristic impedance of the stub section; $Z_C = 1.0$ here, because we are using normalized impedances as discussed in Sec 4.2.

Transformer section T transforms the load impedance to varying values of both resistance and reactance along its length. Hence, for a proper match, the magnitude of the feed-line section F impedance, Z_C, depends on the location of the feed point, and vice versa. This is because the feed-line impedance, Z_C, must equal the resistance component of the line impedance appearing on the transformer line at whatever point the feed line is attached to the transformer. This concept is not generally appreciated, and it is certainly not readily apparent from the usual stub-length and position-indicating graphs appearing in many publications.

Observe that, as we move from diagrams 'b' to 'c', the feed line has moved toward the short-circuited end of the stub. We also observe the resistive component of the line impedance on the transformer section and the Z_O of the feed line decreasing from 1.0 to 0.43 ohms. However, observe also that, when the total length of the transformer section T becomes 90° in length, or λ/4 in diagram 'c', the stub length has gone to zero, and transformer T has evolved into the conventional λ/4 matching transformer. Note that in the case of the λ/4 transformer, the input and load resistances are reciprocally related. From this relationship it can be shown that, when the input and output impedances of practical λ/4 transformers are normalized to the characteristic impedance Z_O of the transformer, the input and output impedances will always be reciprocally related. This illustration indeed demonstrates the remarkable, but simple phenomenon of wave interference in achieving the matching of impedances by introducing complementary reflections to eliminate the reflections resulting from a mismatch.

Sec 4.6 Stub Matching Versus Network Matching

We are now getting closer to understanding how stub-matching principles extend to line-matching-network operation. If we look further into the reflection characteristics of what have generally been considered to be different techniques of matching, a fascinating revelation of the similarity among all of these various techniques emerges. Stub, hairpin, λ/4 series section transformer, transmatch, L network, pi network, and so on, are all in this category. And there is a logical reason for this similarity—these techniques all have one essential ingredient in common—reflections! Reflection and matching are applied in transformers that rely on reflections from the end terminals, where a change in impedance level exists. As I discussed in Chapter 3, any abrupt change in impedance level appears as a discontinuity to the smooth flow of the electromagnetic wave, and results in producing a reflection. The transformer accomplishes the task of matching its input and output impedances by controlling the phase and magnitude of the waves produced by reflection at its boundaries, or end terminals. As a result, all the reflections produced at either end are canceled by those arriving from the other end *(Ref 35, p 58)*. This is what was meant in the reference to "controlled reflections" early in Chapter 1. A corollary to the seeming anomaly of a stub producing a mismatch, instead of a match, is that we match to eliminate reflections, but we can't match without using controlled reflections when different impedance levels are involved.

From Eq 3-1, we know that the ratio between the load impedance Z_L and the impedance Z_C of the line transformer determine the magnitude of a reflection from a mismatched load. To enhance the understanding of the role played by reflections in the process of impedance matching, it is interesting to make two additional observations on the Vector Graph, Fig 3-2. First, either the magnitude of the reflection, ρ, or the SWR, determines the position on transformer section T where the resistance component (the real part[5]) of the transformed line impedance Z = E/I equals the feed line characteristic impedance Z_C on the *unity resistance circle (the R = 1 circle).* This position, we recall, is the matching point, and fixes the length of transformer section T. In making this observation, remember that the diameter of the SWR circle is proportional to the SWR. Thus by tracing along the R = 1 circle we can see that, as the diameter of the SWR circle changes, the point where the R = 1 circle and the SWR circle intersect moves accordingly. A radial line drawn through this intersection point, and extending to the line-length scale len°, indicates the angular distance (T) from the load to the matching point for a given SWR. We recognize this observation as simply the conventional method of using the Smith Chart for determining the stub position when the transformer and feed-line impedances Z_C are equal. But in addition, using a radial indicating line in a similar fashion while tracing along the SWR = 3.0 circle as it intersects the various other resistance circles, we see that for a given SWR the resistance component[5] of the line impedance Z = E/I changes with position along the transformer. These two observations together reveal a flexibility available in the approach to a matching procedure that provides a step toward visualizing the fundamental similarity of the different matching techniques. This flexibility includes the following three conditions, which are explained in more detail later.

1) There is no restriction on the characteristic impedance Z_C of the transformer section T that requires it to be of the same value as the section F feed-line impedance–it can range from low (coax) to high (open-wire line, whichever is to be used).

2) The *length* of the transformer section having a given Z_C can be found which will yield the desired resistance component[5] of the transformer-line impedance that matches the feed-line section F impedance Z_C, which differs from the transformer Z_C. However, the transformer section can have a length that is not limited to the distance from the terminating load to the first point at which the resistance component[5] of the impedance is seen to equal the feed-line Z_C. The transformer can extend from the load to either of the two points where the resistance component[5] is seen to equal the feed-line Z_C on the SWR circle, or any electrical length extending beyond these points by an integral multiple of λ/2. Later I show how the use of matching networks assists in obtaining the required electrical length. This removes *all restrictions* from any specified physical transformer length, that is, from the load (the antenna) all the way to the operating position.

3) Any device that will provide a reactance of the correct value can perform the action of the stub portion. This can be either by a lumped-constant component, or by a separate line section of any reasonable value of Z_C of the proper length to obtain the required value of reactance. The *electrical* length of the stub is always

directly related to its reactance. Now we again return to wave or reflection mechanics. We will see in those terms how matching, obtained by the various techniques recited previously, is described by three parameters: transformer impedance, transformer length, and stub reactive elements.

In our earlier example using the stub technique, the magnitude ρ of the reflections appearing at each end of the *transformer* section was the same, 0.5, or an SWR of 3:1, for the load at one end and the stub at the other. In other words, the magnitudes of each complementary mismatch were identical. For the present, we will retain the characteristic impedance $Z_C = 1.0$ for the entire stub-transformer line section. But based on conditions 1 and 2 above, we may change the feed-line impedance as conditions dictate. Consider now the effect of increasing the length of the transformer section and shortening the stub section in accordance with Eq 4-1. For example, while referring to Fig 4-2C and the Vector Graph, Fig 3-2, let us move the feed-line entry point farther away from the load, from len° = 30° to len° = 60° This increases the transformer length to 60° and reduces the stub length to 26.6°. Now on the Vector Graph observe the radial line extending from the len° = 60° point (where $\theta = -120°$) through point B, where the SWR = 3.0 circle and the R = 0.43 circle intersect. Observe that the corresponding movement along the SWR circle results in a change in the resistance component at the feed point, from R = 1.0 to a new resistance, R = 0.43.

Now recall from the earlier statement that the complementary mismatch relation holds constant wherever the feed line is attached. A feed line having a characteristic impedance $Z_C = 0.43$ will be perfectly matched when attached at the len° = 60° point. The complex voltage reflection coefficient of the load-mismatch is now read as $\overline{\rho}_E = 0.5 \angle{-120°}$ at point B. The magnitude is the same as before, but the phase angle is *larger* because we are farther from the load. And applying the complementary-mismatch principle, we see that the complex voltage reflection coefficient of the stub mismatch becomes $\overline{\rho}_E = 0.5 \angle{+120°}$, as read at point C. So we ask the question, how does this new combination produce a canceling wave having the same magnitude as before? Recall that previously, when the line-characteristic impedance of sections F, S and T were all $Z_C = 1.0$ as in Fig 4-2B, the canceling reflection was generated by the stub alone. This happened because no line-junction mismatch existed between the feed line section F and the transformer T. But now that the impedance of the feed line differs from the impedance of the transformer, we have an additional discontinuity at the feed point, which also generates a reflection. And the *shorter* (series) stub portion now generates a reflection which is smaller than when all the line sections had a $Z_C = 1.0$. The magnitude of the stub-reflection is reduced by the amount of the reflection presently being generated by the feed-line-to-transformer mismatch. Thus, by the complementary mismatch principle, the resulting canceling wave still retains the correct magnitude and phase to cancel the load-mismatch reflected wave at this new feed point on the transformer. This canceling wave is evidently generated by the *combined discontinuities* of both the differing line impedances at the feed-line junction, and of the stub with the corrected length. We have thus matched a feed line of $Z_C = 0.43$ to a load of $Z_L = 3 + j0$ through a transformer of $Z_C = 1.0$.

Using this same type of reasoning, to obtain a match for feed lines of higher impedance, we may conversely shorten

Table 4-1
Matching Characteristics with Various Transformer- and Stub-Section Lengths

				Angle of Reflection Coefficient			
				Voltage		*Current*	
Transformer Length len°	*Stub Length S_L°*	*Resistance Component R*	*Stub Reactance jX*	*Load Mismatch θ°*	*Stub & Line Mismatch θ°*	*Load Mismatch θ°*	*Stub & Line MisMatch θ°*
0	0	3.0	0.0	0	0	180	180
10	48.2	2.42	+1.12	-20	+20	+160	-160
22.5	52.9	1.38	+1.32	-45	+45	+135	-135
30	49.0	1.00	+1.15	-60	+60	+120	-120
45	38.7	0.60	+0.80	-90	+90	+90	-90
52	33.0	0.50	+0.65	-104	+104	+76	-76
60	26.6	0.43	+0.50	-120	+120	+60	-60
67.5	19.8	0.38	+0.36	-135	+135	+45	-45
90	0	0.333	0.0	180	180	0	0

the transformer section and change the stub section, according to the tangent relation in Eq 4-1. To accomplish this we position the feed-line entry point where the real part of the line impedance Z = E/I in the transformer T has been transformed to the value of the feed-line impedance Z_C that we wish to use. We then adjust the stub length to cancel the line reactance. As explained above, the resistance circle which is intersected by the SWR circle for a given transformer length indicates the real part of the of the feed point impedance. This is also the Z_C value of the feed line that will be perfectly matched when connected at that feed point on the transformer. The data presented in Table 4-1, taken from points along the SWR = 3.0 circle on the Vector Graph, show a few selected transformer-length examples, and pinpoint some of the pertinent information for clarity. Notice especially that the real part of the feed-point impedance decreases as the transformer length increases. It is interesting to discover that when the feed point goes beyond the len° = 45° position, the θ angles of the voltage- and current-reflection coefficients pass through 90° from opposite directions. The result of this is that their respective resultants shift 180°. Thus, the resultant θ angles of reflection coefficient interchange. The resultant angle θ of the voltage reflection coefficient now becomes 180° and the resultant current angle θ becomes 0°. This means that the effective reflecting termination at the match point shifts from an open circuit to a short circuit when the feed line is attached more than 45° away from the E_{MAX} position (at len° = 0° on the transformer).

Consider now the effect of increasing the length of the transformer section until the reactance component of the line impedance disappears by itself without requiring a stub to cancel it. The Vector Graph shows this condition to occur at len° = 90°. At this point the reflected voltage wave from the load mismatch is exactly 180° out of phase with the forward wave, and therefore no reactance component is developed. The resistance component[6] of the line impedance at this point is 0.33 × ZC, as shown on the chart. Hence, we may now connect a feed-line section F having impedance Z_C = 0.33 chart ohms at this point and obtain a perfect match, as shown in Fig 4-2D. No reflections appear on the 0.33-chart-ohm feed line. Why? Again, because of the canceling reflections in the transformer section! Note the present length of the transformer section — 90°, or λ/4. The transformer section alone is now using the *entire length* of the line section, and the stub section has disappeared.

Note that simply by moving the feed point along the transformer to the point where the line reactance vanishes, the resistance component[5] becomes Z_C/SWR = 1/3, and we slip smoothly from the stub form into the series λ/4 transformer form of matching. You can see what's happening by observing the numbers in Table 4-1. Remember, the characteristic impedance of the transformer is still $Z_C = 1.0$, which has become the geometric mean between the input and output impedances that it is matching, $\sqrt{0.33 \times 3.0}$. From inside the transformer, the impedance level looking toward the input terminals is stepped down 3:1 from 1.0, giving us short-circuit reflection behavior, just as the output impedance is stepped up 1:3 from 1.0, for open-circuit reflection behavior at the output terminals. It is therefore evident that a λ/4 transformer section of line having a Z_C equal to the geometric mean of its two end-terminal impedances has equal mismatches at both ends, and thus produces reflections of equal magnitude at both ends. These reflections from each end cancel each other at the junction of the feed line and the transformer input terminals, as waves reflected at the output load mismatch return to the input junction exactly 180° out of phase with the wave reflected at the input mismatch. This is because the wave reflected from the load mismatch has traveled 90° from the input point to the load mismatch, and an additional 90° in returning to the input. (See Chapter 23)

To clarify further what is happening here, recall that previously when the feed-line section F and transformer T were of equal impedance $Z_C = 1.0$ (Fig 4-2B), the canceling reflected wave was generated entirely by the stub mismatch. In the present case, where the stub length is zero and the transformer length is λ/4, the canceling reflected wave is generated entirely by the 3:1 mismatch at the junction of the feed line and the transformer. The voltage reflection coefficient angle of the wave reflected from this feed-line-junction mismatch is $\theta = 0°$ referenced to the feed-line Z_C, because the Z_C of the transformer is greater than the Z_C of the feed line. However, after the waves reflected from both the load- and input-junction mismatches have joined at the input of the transformer to cancel one another, the input-junction reflection no longer travels toward the generator, but is re-reflected back into the transformer toward the load in the same manner as with the previous stub-reflected wave. Therefore, referenced from within the transformer, the reflection coefficient angle of the feed-line-junction reflection is $\theta = 180°$, as indicated in Table 4-1. The matching point with this configuration is at the input terminals of the λ/4 transformer section. Hence, it is evident that the above discussion has explained the reflection mechanics involved in the general impedance-matching action of all series λ/4 transformer matching sections.

Chapter 12 explains why the λ/4 series transformer is an impedance *inverter for any complex terminating load.* It is not restricted to purely resistive loads.