A Tutorial Dispelling Certain Misconceptions Concerning Wave Interference in Impedance Matching

Walt Maxwell responds to Steven Best's description of how reflected waves act at mismatched boundaries.

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In a three-part article appearing in *QEX,*[1] Dr. Steven R. Best, VE9SRB, wrote about the wave mechanics involved in impedance matching. In that article, he disputes the treatment of wave mechanics appearing in the writings of Slater[2] and Alford,[3] in all editions of "*Reflections,*"[4, 5, 7] and my article in *QEX.*[6]

The errors in Steve's article follow from an invalid premise. This detailed tutorial is presented to describe and correct that. The invalid premise is revealed in his Eqs 6, 7, and 8 of his Part 1.

All three of these equations are invalid because Steve misinterpreted the universally known equations for determining the amplitude of voltage, E, of the standing wave as the equations for determining forward voltage V_{FWD}. The two separate voltage values, voltage E of the standing wave, and forward voltage V_{FWD}, resulting from these two equations differ significantly. As we know, the amplitude of the voltage standing wave is the phasor sum of the forward and reflected voltages, which varies alternately between maximum and minimum with varying distance along the line. However, on lossless lines, the amplitude of forward voltage V_{FWD} is constant throughout the entire length of the line. The forward voltage can be determined using the correct expression

$$V_{FWD} = \sqrt{P_{FWD} \times Z_0} \quad \text{(Eq 1)}$$

where P_{FWD} is the total forward power, and Z_0 is the characteristic impedance of the line. That expression is proven correct through the following mathematical exercise: Forward current

$$I_{FWD} = \sqrt{\frac{P_{FWD}}{Z_0}} \quad \text{(Eq 2)}$$

and $V_{FWD} \times I_{FWD} = P_{FWD}$. A numerical example presented later verifies it. In addition, contrary to Steve's assertion, P_{FWD} equals the sum of the source and reflected powers,

[1]Notes appear on page 50.

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not the voltages, which will also be proved later. The derivation of the familiar equations for determining standing-wave voltages, *E,* along the line that Steve misinterpreted to obtain his Eqs 6, 7, and 8 can be found in Johnson,[8] Sec 4.3, pp 98 and 99.

In Steve's Eq 6 in Part 1, the expression shown below for forward voltage V_{FWD}, is invalid because the left-hand term of the equation V_{FWD} does not equal the sum of the terms on the right-hand side of the equation.

$$V_{FWD} = V1 + V1\left(\rho_S \rho_A e^{-2\gamma L}\right) + V1\left(\rho_S{}^2 \rho^2{}_A e^{-4\gamma L}\right) + V1\left(\rho_S{}^3 \rho^3{}_A e^{-6\gamma L}\right) + \ldots \quad \text{(Eq 3)}$$

(Steve's Eq 6)

The terms on the right-hand side of his Eq 6 yield the voltage E of the standing wave, not the forward voltage V_{FWD}. His Eqs 7 and 8 are also invalid for the same reason. So, let's demonstrate that his Eq 8 is invalid, which will also prove his Eqs 6 and 7 invalid.

$$V_{FWD} = V1\left(\frac{1}{1 - \rho_S \rho_A e^{-2\gamma L}}\right) \quad \text{(Eq 4)}$$

(Steve's Eq 8)

$V1$ = source voltage,

ρ_S = reflection coefficient looking back into the source, and

ρ_A = voltage reflection coefficient of the load (antenna) mismatch.

First, using a lossless line, we let $\rho_S = 1$, and $\rho_A = 1$. In this case the denominator is zero, thus $V_{FWD} = \infty$, which is an impossibility. Second, we change ρ_A to 0.5 resulting from a 3:1 mimatch at the antenna. In this case the denominator is 0.5 and $V_{FWD} = V1 \times 2$—also impossible. We will show later that under the conditions just stated, the forward voltage increase factor caused by the integrated reflections added to the source voltage is 1.1547. Therefore, the forward voltage in this case is $V_{FWD} = V1 \times 1.1547$. Consequently, we have shown that Steve's Eqs 6, 7, and 8 do not yield the correct forward voltage V_{FWD}, as he claims. They only yield the standing wave voltage E, where L in the exponent of Eq 8 specifies the position on the line.

As we'll also prove later, forward current I_{FWD} equals source current $I1 \times 1.1547$. Thus, since forward power $P_{FWD} = V_{FWD} \times I_{FWD}$, the power increase factor is $1.1547^2 = 1.3333$ when $\rho_A = 0.5$, which is a well-known result. The derivations of the voltage, current and power increase factors, which prove this treatment to be true, will also be presented.

VE9SRB's Fallacy

At the opening of his Part 3, Steve calls total re-reflection a fallacy. He states: "*…it is a misconception that total re-reflection of reflected power occurs at a match point as the result of an impedance match being established there."*

Then, after attempting to prove total re-reflection doesn't exist, he concludes, "*…the concept of total re-reflection is inconsistent with generally accepted transmission-line theory, basic circuit theory, and network theory."* Then from his summary, *"It was demonstrated that the concept of total power re-reflection is inconsistent with both transmission-line and circuit theories."*

On the contrary, his mathematical treatment did not demonstrate any inconsistency in the concept of total re-reflection. And an additional claim in his conclusion is also untrue: *"A total re-reflection of power at the match point is not necessary for the impedance match to occur."* His conclusion is untrue because, as we will show, without total re-reflection the impedance match would not be accomplished. Furthermore, Steve has it backward: the impedance match *results* from total re-reflection, not the other way around.

To show that total re-reflection is not a fallacy, let's first examine the implications of his Eq 2, Part 3,

$$P_{FWD} = P_{DEL} = \frac{1}{1 - k^2 |\rho A|^2} \quad \text{(Eq 5)}$$

that he states, "*… develops the relationship between delivered and forward power.*" He continues, writing: *"Eq 2 is independent of whether an impedance match occurs at the T-network input…. Eq 2 does not support the concept of total re-reflection of power when an impedance match is established at the T-network input."*

Of course it doesn't, because a vital term is missing in his Eq 2: the reflection term ρ_S, which determines the reflective condition seen by the reflected waves on return to the network input, and thus determines the amount of energy in the reflected waves will be re-reflected. Consequently, with term ρ_S missing, both the amount of re-reflection and the increase in forward power are undeterminable using his Eq 2. However, the complete *correct* equation,

$$P_{FWD} = \frac{1}{1 - k^2 |\rho_S \rho_A|^2} \quad \text{(Eq 6)}$$

derives the forward-power increase factor based on the reflection coefficients at *both* ends of a transmission system, not just at the load end. The term k^2 represents the decimal value of power lost to line attenuation, the term ρ_A represents the reflectivity of the load, or antenna mismatch, and the term ρ_S represents the reflectivity to the power in the reflected waves on their return to either the network input or the source. When $\rho_S = 0$, the reflected power is totally absorbed in the source, thus *no* re-reflection and *no* increase in forward power, as in the classical generator. On the other hand, when $\rho_S = 1$, which is the prevalent condition at both the network input and at the source when adjusted to deliver all the available power, there is *total* re-reflection and maximum increase in forward power, indicating an impedance match at the input. *Without total re-reflection there would be no match.* This issue appears to be the crux of the problem concerning total re-reflection: the wave activity that occurs at the network input, the matching point in the network. How the reflective condition $\rho_S = 1$ to reflected waves is established at the network input will be explained later while discussing the establishment of open and short circuits by wave interference.

Steve created another misconception: that the forward traveling source and re-reflected voltage waves are vectorial in nature. He asserts, **"***The forward traveling voltage in the transmission line is a complex phasor that can be written in the general form of a vector* $\mathbf{V} = x + jy = V\angle\theta$." Thus, citing his Eq 9, Part 3,

$$P_F = \frac{|V_1 + V_2|^2}{Z_0} \quad \text{(Eq 7)}$$

he asserts incorrectly that phasors V_1 and V_2, are added vectorially to obtain the forward power in a mismatched transmission line, where P_F = forward power, and V_1 and

V_2 are the phasors of the source and re-reflected voltages, respectively.

His Eq. 9 is an invalid expression for use on a line with re-reflections, because in a lossless line, the relative phase angles for both V_1 and V_2 are θ = 0° everywhere along the line. Keep in mind that on a lossless line, voltage and current in the forward wave are always *in phase*, while voltage and current in the reflected wave are always 180° *out of phase*. Consequently, reflected power is real, not reactive power. To be reactive, the phase relation between voltage and current would need to be other than 0° or 180°. As will be explained in the section discussing open and short circuits established by wave interference, the wave action on re-reflection brings *all* re-reflected voltages and currents into zero phase relative to the source waves, regardless of their phase relationship prior to re-reflection. Consequently, Steve's lengthy dissertation on the different values of forward power that could result from various θ angles of V_1 and V_2, and Eq 9 itself, are irrelevant. ***Voltages V_1 and V_2 are never at other than 0° phase relative to the source phase at any time on lossless lines.*** Although V_2 is the resultant of two reflected waves whose magnitudes are equal, with equal but opposite phase angles, phase angle θ of the *re-reflected* resultant voltage V_2 following re-reflection is always zero on lossless lines.

An additional point that Steve failed to recognize is that the addition of the source and reflected voltages establishes the standing wave, not the forward voltage, which has a constant value with zero phase everywhere along the line. So let's examine what happens when we use his Eq 9 to perform this addition.

Steve's Fig 1, Part 3 shows a 100 W, 50-ohm source feeding a T-network connected to an antenna of impedance Z_A = 150 ohms through a lossless 50-ohm, 1 wavelength transmission line. In his example, source voltage V_1 = 70.71; forward power incident on the antenna, P_{FWD} = 133.33 W; and power reflected P_{REF} = 33.33 W. For P_{FWD} = 133.33 W, the total forward voltage V_{FWD} must be 81.65 V, and with reflection coefficient $\rho_A = 0.5$, reflected voltage V_2 = 40.825 V. In addition, the source current is I_1 = 1.414 A, total forward current I_{FWD} = 1.633 A, and reflected current I_{REF} = 0.8165 A. Each of these values is correct; the article also states correctly that the re-reflected voltage must equal the reflected voltage.

However, Eq 9 is invalid because it states *incorrectly* that re-reflected voltage V_2 adds directly to source voltage V_1 to establish forward power—*it does not*. Steve does state correctly that the in-phase 40.825 V re-reflected voltage V_2 and 70.71 V source voltage V_1 cannot be added together such that the total voltage will be 81.65 V. Whoa! This should have been a clue that source and reflected voltages add only to establish the voltage *standing wave*, *not forward voltage* V_{FWD}, because $V_{FWD} \times I_{FWD}$ = 133.33 W. We prove his Eq 9 for forward power, P_{FWD} erroneous by showing that

$$P_{FWD} = \frac{|V_1 + V_2|^2}{Z_0} = \frac{|70.71 + 40.825|^2}{50} = 248.8 \text{ W} \quad \text{(Eq 8)}$$

not the correct 133.33 W. However, it must be said that Eq 9 is valid when there are *two separate and independent sources*, such as two generators. But it must also be kept in mind that the power in the reflected waves is delivered by only *one* source, the transceiver. Apparently, this anomaly, and knowing that the relationship between forward and reflected voltages is vectorial, are what led to the erroneous concept of a vectorial relationship between the forward and re-reflected voltages.

We'll now reveal the correct mathematical expression for obtaining forward power from V_1 and V_2 that proves source and reflected powers do indeed add to establish the total forward power. The universally known reciprocally related equations for determining either forward power P_{FWD} or delivered power P_{DEL} in a mismatched transmission line are:

$$P_{FWD} = \frac{|V_1|^2 + |V_2|^2}{Z_0} = \frac{|70.71|^2 + |40.825|^2}{50} = 133.3 \quad \text{(Eq 9)}$$

and

$$P_{DEL} = \frac{|V_{FWD}|^2 - |V_2|^2}{Z_0} = \frac{|81.65|^2 - |40.825|^2}{50} = 100 \quad \text{(Eq 10)}$$

where V_1 = source voltage, V_2 = reflected voltage, and V_{FWD} = forward voltage. The terms in the numerators are *power terms*, thus either of the two equations above indicates that forward power equals source power plus reflected power. Steve disagrees that forward power equals source power *plus* reflected power, so it's ironic that he somehow agrees that power delivered to the load is equal to the forward power *minus* the reflected power. See *Reflections*, Chapter 3, Eq 3-9, and Johnson,[8] p 150, Eqs 6.14 thru 6.17 for the derivation of those equations. To verify that the correct forward power is 133.33 W, we'll now use an alternate method to determine that value. Using the standard equation for calculating the forward-power increase factor from the earlier example,

$$\frac{1}{1 - |\rho_S \rho_A|^2} = 1.333 \quad \text{(Eq 11)}$$

where $\rho_S = 1$ and $\rho_A = 0.5$, with source power 100 W, we have shown that the total forward power is 133.33 W on a lossless line.

So let's carry the math in this exercise a little further to provide additional proof that the exercise has been performed correctly. From Ohm's Law, we know that when forward power on a 50-ohm line is 133.33 W, the forward voltage must be 81.65 V and the forward current must be 1.633 A. Also from Ohm's Law, when resistance is constant in a circuit, voltage equals the square root of power. So we'll now take the square root of the P_{FWD} equation to establish both the forward voltage and current increase factor,

$$V_{IF} = I_{IF} = \sqrt{\frac{1}{1 - |\rho_S \rho_A|^2}} = \sqrt{1.333} = 1.1547 \quad \text{(Eq 12)}$$

where, as above, $\rho_S = 1.0$ and $\rho_A = 0.5$. As before, source voltage V_1 = 70.71 V, and source current I_1 = 1.414 A. Thus $V_1 \times V_{IF}$ = 70.71 V × 1.1547 = 81.65 V = V_{FWD}, the forward voltage determined earlier, and $I_1 \times I_{IF}$ = 1.414 × 1.1547 = 1.633 A, I_{FWD}, the forward current. Now, V × I = P, 81.65 V × 1.633 A = 133.33 W, thus proving our case.

Note that the forward voltage V_{FWD}, 81.65 V, is an increase of only 10.94 V from the 70.71 source voltage V_1, and the forward current 1.633 A is an increase of only 0.219 A from the 1.414 A source current. However, these small increases in voltage and current represent the corresponding increase in forward power from 100 W to 133.33 W, proving that it is the reflected power adding to source power that establishes the forward power, *not the addition of the reflected voltage to source voltage.*

We can now show that the steady-state forward voltage V_{FWD} *plus* the reflected voltage V_2 establishes the maximum of the voltage standing wave, (81.65 V + 40.82 V = 122.47 V), and V_F *minus* V_2 establishes the minimum of the voltage standing wave (81.65 V – 40.82 V = 40.82 V). Note also that the ratio of the max and min voltages of the standing wave is 122.47 ÷ 40.82 = 3.0, verifying that the values above are correct. We have thus disproved the assertion that adding source and reflected powers to establish the forward power is a fallacy.

Open and Short Circuits Established by Wave Interference

In another quote from Part 3 of Steve's article he asserts that open and short circuits to reflected waves *cannot* be established by the wave interference involved in impedance matching. He writes, *"For a total re-reflection of voltage, current, or power to occur at a T-network, transmission line theory requires that the physical or measurable impedance seen looking rearward into the matching network be a short circuit, open circuit or purely reactive. Since this generally would not be the case with a practical T-network impedance matching circuit, the concept of total power re-reflection contradicts this fundamental aspect of transmission-line theory."*

That is fundamentally incorrect and also disputes Slater[2] and Alford.[3] Those authors have shown that a physical short or open circuit is not what accomplishes total re-reflection of the reflected waves. Wave interference between two sets of reflected waves traveling in the same direction within a transmission line or network that are conjugately related at the matching point establish either a one-way *short* circuit or a one-way *open* circuit to the rearward-traveling reflected waves. We'll now show how two sets of reflected waves traveling in the same direction are established in the impedance-matching process, and how they also establish the one-way short or open circuit that prohibits any further rearward travel of the reflected waves, in other words, total re-reflection at the match point.

In general, the impedance-matching process involves three harmonically related traveling waves arriving at the match point: a forward wave delivered by the source (Wave 1), and two conjugately related rearward-traveling reflected waves (Waves 2 and 3) developed by two conjugately related physical discontinuities. Wave 2 is the wave reflected from a mismatched line termination that requires cancellation, and Wave 3 is the canceling wave reflected by the matching device at the match point, its input connection to the source line. Because of their conjugate relationship, Waves 2 and 3 are mirror images of each other.

First, with waves traveling in opposite directions on a transmission line, we know that reflected waves pass through forward waves unimpeded, and the interference between them establishes only a standing wave—no open or short circuits are established. However, when two sets of voltage and current reflected waves are traveling in the *same* direction and are conjugately related at the matching point in an impedance matching device, the interference between these two sets of reflected waves establishes either an open or short circuit at the matching point. Whether an open or short circuit is established depends on the boundary conditions of the mismatched load and the distance from the load to the matching point. When the match point at the normalized unity-resistance point using a *series* stub on the line occurs within the first quarter-wavelength from the load, an open circuit to reflected waves occurs at the match point when the resistive component R of load $Z_L > Z_0$. A short circuit to reflected waves occurs when the resistive component R of load $Z_L < Z_0$. The reasons for these phenomena will become clear as we proceed.

In learning how one-way open and short circuits are established through wave interference, it is helpful to first understand what happens to an electromagnetic field on encountering a physical open or short circuit. It is universally known that when an electromagnetic field encounters an open circuit, the electric field (or voltage wave) is totally reflected with 0° change of phase, and the magnetic field (or current wave) is totally reflected with a 180° change of phase. Conversely, when an electromagnetic field encounters a short circuit, the electric field (or voltage wave) is totally reflected with 180° change of phase, and the magnetic field (or current wave) is totally reflected with a 0° change of phase. *It is of vital importance to the issue of total re-reflection to understand that these relationships are reciprocally related.* Consequently, when the resultant voltage and current angles established by wave interference are 0° and 180°, respectively, an open circuit to the reflected waves is established. Conversely, when the resultant voltage and current angles are established at 180° and 0°, respectively, a short circuit to the reflected waves is established. Thus, when either an open or a short circuit is established at the matching point by wave interference between the two sets of conjugately related reflected waves traveling rearward, the direction of the voltages, currents, and energies in both sets of reflected waves is reversed. That results in total re-reflection of the reflected waves.

Let us now determine why open or short circuits are developed by wave interference. From King,[9] we know that voltage and current traveling along the line can be represented by individual generators placed at any point along the line. Those generators are called "point generators." For the purpose of analysis, a point generator is an impedance-less EMF that can represent or replace the voltage and current on the line equal to the voltage and current actually appearing at that point on the line, without disturbing the wave action on the line.

To simulate and analyze interference between two waves of equal magnitude and opposite phase traveling in the same direction, such as the two sets of reflected waves generated by the load mismatch and the stub mismatch, we can connect two point generators together in either of two different configurations. Each generator replaces the voltage and current of each individual wave at the point of interference, the match point. In the first configuration, the two generators are connected *in phase*. Because their voltages are equal and in phase, the differential voltage is zero, resulting in no current flow. This connection is equivalent to an *open* circuit between the generators. In the second configuration, the generators are connected with their terminals reversed. Their voltages are now *in opposite phase* at the interference point and the resulting voltage is the *sum* of the voltages delivered by each generator; i.e., twice the voltage of each generator. This connection results in a *short* circuit between the two generators.

Identical wave-interference phenomena establishing a short circuit also occur in free space in the same manner as in guided-wave propagation along transmission lines. For example, when the fields emanating from two radiators in an array of antennas are of equal magnitude and 180° out of phase at a point in space, a virtual short circuit is established by destructive wave interference, resulting in a null in the radiation pattern at that point. Following Poynting's Theorem, the energy in the combined fields propagating is reversed in direction at that point; and with the constructive interference that follows, that energy adds to that in the fields propagating in the opposite direction, thus achieving gain in the that direction.

Stub matching on a mismatched transmission line provides an elegant model to illustrate the mechanism in wave interference that establishes one-way open or short circuits to reflected waves on transmission lines.[10] When placed at the matching point on the transmission line, the stub is designed to introduce a mismatch. That produces a canceling reflection having the same magnitude as the reflection from the mismatched load terminating the line, but with the opposite phase angle. The following example illustrates this condition.

Assume a lossless line of Z_0 = 50 ohms terminated in a pure 150-ohm resistance, Z_L, yielding a 3:1 mismatch for a voltage reflection coefficient $\rho = 0.5$. An appropriate point to place a series stub on the line is at a normalized point of unity resistance. When $\rho = 0.5$, the first point of unity resistance appears at 30° rearward from the load toward the generator. The line impedance at this point is $50 - j57.7$ ohms. Traditionally, a $+j57.7$-ohm inductive reactance inserted in series with the line at this point cancels the $-j57.7$-ohm capacitive line reactance, achieving a Z_0 impedance match at this point, leaving a 1:1 flat line between the inductance and the source, and 3:1 mismatch on line between the inductance and the load. However, it makes no difference whether a series lumped inductor or a series stub, each presenting $+j57.7$ ohms, supplies the inductive canceling reactance. Either one establishes the impedance match.

But there is more occurring here than a simple cancellation of reactances. The matching process is actually performed by the mechanics of wave interference at the matching point that generates either an open or short circuit to the reflected waves, preventing them from traveling rearward past the stub point. To examine the wave action, we focus mainly on the reflection coefficients of the reflected voltage and current waves that appear at the stub point, and to a lesser extent of those at the load, as shown below, where voltage and current coefficients are indicated by V and I, respectively. The remaining nomenclature is self-explanatory.

$\rho_{VLoad} = 0.5$, $\theta = 0°$
$\rho_{ILoad} = 0.5$, $\theta = 180°$
$\rho_{Vline\ at30°} = 0.5$, $\theta = -60°$
$\rho_{Vstub} = 0.5$, $\theta = +60°$ Resultant $\angle\theta_{RV} = 0°$
$\rho_{Iline\ at30°} = 0.5$, $\theta = +120°$
$\rho_{Istub} = 0.5$, $\theta = -120°$ Resultant $\angle\theta_{RI} = 180°$

Note that the magnitudes of all reflections are equal at $\rho = 0.5$. Note also that the voltage angles for the line and stub equal but of opposite sign, as are the corresponding current angles. In addition, note that the resultant angles of the voltage and current are $\theta_{RV} = 0°$, and $\theta_{RI} = 180°$, respectively. We learned from the earlier detailed discussion above, that when the magnitudes of the reflections are equal, and the resultant voltage and current angles established by wave interference are 0° and 180°, respectively, as they are in this case, an open circuit to the reflected waves is established. Consequently, the open circuit prevents any further rearward wave travel beyond the matching point—*total re-reflection*—resulting in a Z_0 match at the stub point.

An Analysis of Steve's T-Network Tuner

Refer to Steve's section, *"The T-Network Tuner,"*[1] in Part 3, where he analyzes the wave actions in a system comprising a 50-ohm, 100 W source; a lossless tuner feeding a 1.20 λ, transmission line; and a 50-ohm lossless line terminated in a 150-ohm load (the antenna). As stated earlier, the load mismatch at the antenna establishes reflection coefficient $\rho = 0.5$, SWR = 3:1. The line then transforms the 150-ohm load impedance to impedance $Z_L = 18.213 - j14.237$ ohms, $\rho = 0.5$ at the main line input connected to the output of the source. The object is to match Z_L to the 50-ohm source. However, in examining the analysis, we find errors, such as:

1) the mathematical model is inconsistent with realistic wave activity in the steady state,

2) the setup is treated using mostly initial-state conditions, not the steady state,

3) reflected voltages are added incorrectly, as described earlier, to determine forward power,

4) forward and reflected powers determined from his Eqs 5 and 6 are using either incorrect voltages or incorrect values of Z_0.

The mathematical model used in Steve's analysis begins correctly, showing 133.33 W of incident or forward power arriving at the antenna and 33.333 W reflected because of the 3:1 mismatch at the antenna. Using that model, however, his analysis shows *incorrectly* that useful re-reflection occurs at the output of the network and none at the input, while in fact, *all useful* re-reflections occur at the input, the matching point, as we will prove.

Steve's article states that on encountering reflection coefficient $\rho = 0.5$ at the tuner output, 25 percent (8.33 W) of the 33.333 W reflected at the antenna is re-reflected back to the antenna and 75 percent (25 W) "…[is] the level of rearward power delivered back into the T network." He further states: "… the rearward-traveling voltage delivered back into the main transmission line, V_{BACK} is $-32.940 + j12.843$ V." It is significant to note that after traveling rearward through the network, the 25 W of the 33.33 W reflected from the antenna mismatch–which *must* arrive at the network input–is totally ignored. However, Steve then writes: *"This rearward-traveling voltage is the exact negative of the reflected voltage created at the input of the T network* (+32.940 – $j12.843$ V). *Therefore the total steady-state rearward-traveling voltages within the main transmission line is 0 v. This cancellation of all rearward-traveling waves is the mechanism that causes the effective steady-state input impedance to be 50 ohms at the input to the T network."*

Cancellation of *all* rearward traveling waves? This is a huge stretch from simply canceling the reflected *voltages*. Our earlier explanation of the matching process proves that it is not that simple. Does canceling the rearward-traveling waves also cancel the energy carried in those waves? In addition to not accounting for the 25 W of reflected power, he also neglected the rearward-traveling *current*, which is a component of the 25 W that returned from the tuner output mismatch to the tuner input. Consequently, he did not incorporate the resultants of the reflection coefficients of both voltage *and current* at the network input to determine their necessary participation in the wave interference process that accomplishes total re-reflection. Failure to consider the currents involved, and believing that re-reflection does not occur at the network input, led to an inappropriate development of the equations with erroneous results.

Now let's analyze the T network from Steve's article, using the same network parameters as shown below, but with a different mathematical model that will show that *all* re-reflections pertinent to impedance matching occur at the network input. C_1 and C_2 are the input and output capacitors, respectively.

Beginning at the input of the network, recall that the input impedance Zin = 117.810 – *j*57.061 when load impedance Z_{LOAD} = 50 ohms, verified by the equations below.

$X_{C1} = -j269.496$, $X_L = j104.216$, $X_{C2} = -j150.146$, $Z_0 = 50$

$$Zin = \frac{1}{\dfrac{1}{Zload + X_{C2}} + \dfrac{1}{X_L}} + X_{C1} \qquad Zin = 117.809 - j57.061$$

(Eq 13)

Initially the source delivers 100 W into impedance Z = 50 + *j*0 at the input of the main line connecting the source to the network. Source voltage V_S = 70.71 V. On encountering the initial impedance Zin = 117.809 – *j*57.06 ohms at the network input, where, ρ_{IN} = 0.466 – *j*0.182 (| ρ | = 0.5, SWR = 3:1), 75 W enter the network and 25 W are reflected toward the source. The initial reflected voltage equals $V_S \times \rho_{IN}$ =32.94 – *j*12.843 V. On reaching the source, the 25 W reflected rearward from the network input creates an initial 3:1 mismatch between the source and the line input, thus reducing the initial source delivery to 75 W. On reaching the steady state the impedance at the network input changes to Z_{IN} = 50 + *j*0, thus the source returns to delivering 100 W, and the 25 W initially reflected toward the source is now transmitted through the network in the forward direction, along with the original 75 W. However, on reaching the steady state, the reflected voltage increases to 38.036 – *j*14.830 V. As explained in the earlier math example, we showed that in the steady state the forward voltage increased by the forward-increase factor 1.1547 from 70.71 V to 81.65 V. However, Steve states incorrectly that "*…the total voltage at the network input is equal to the sum of the source and reflected voltages, 103.651 – j12.843 V.*" These voltages *cannot* be added directly, as we proved earlier. However, we will also see that in the steady state the voltage reflected at the input will have increased to 38.036 – *j*14.830 V, only to be canceled later by its conjugate returning from the load mismatch as the reflected power becomes re-reflected.

Steve's account of the wave actions described in his analysis is incorrect for the same reasons that the steady-state voltage created at the network input is not +32.940 – *j*12.843, but is really +38.036 – *j*14.830. Consequently, it is evident that the voltage returning from the antenna is *not* the exact negative of what he believes is reflected at the network input. Instead, it is its *conjugate*. This same oversight also resulted in using an incorrect mathematical model of the system.

As explained earlier and in references,[2,4,6,7,9] undesired reflections from a load mismatch are canceled by the conjugately related mirror-image reflections generated by the mismatch established by the matching device; in this case, the T network. We'll now show how the conditions described above have set the stage for total re-reflection at the input of the network.

Using terminology set forth earlier, Wave 1 is the source wave, Wave 2 is the wave reflected from the load mismatch (the antenna), and Wave 3 is the canceling wave established by reflection of Wave 1 at the network input. The voltage and current of Wave 1 initially see impedance Z = 117.810 – *j*57.060 ohms at the network input, establishing reflection coefficients of Wave 3: ρ_{Vin} = 0.466 – *j*0.182 = 0.5, θ_{Vin} = –21.333°, and ρ_{Iin} = 0.5, θ_{Iin} = +158.666°, respectively. Also, on arrival rearward at the network input, Wave 2 "sees" impedance Z = 117.810 + *j*57.060 ohms, the conjugate of the impedance seen by Wave 1. Wave 2 thus establishes reflection coefficients ρ_{Vref} = 0.466 + *j*0.182 = 0.5, θ_{Vref} = +21.333°, and ρ_{Iref} = 0.5, θ_{Iref} = –158.666°. Note that coefficients of Wave 2 are mirror images, or conjugates of those of Wave 3, because the impedances appearing at the network input from opposite directions are conjugates of each other. Consequently, steady-state voltages and currents at the network input are:

Wave 2 = 38.036 + *j*14.83 V = 40.825 V, θ_{V2} = +21.3°,
Wave 3 = 38.036 – *j*14.83 V = 40.825 V, θ_{V3} =–21.3°
…resultant $\angle\theta_{RV}$ = 0°
Wave 2 = 0.7607 – *j*0.2966 A = 0.8165 A, θ_{I2} = –158.666°,
Wave 3 = 0.7607 + *j*0.2966 A = 0.8165 A, θ_{I3} = +158.666°,
resultant $\angle\theta_{RI}$ = 180°.

Note that the voltage magnitudes are equal and their resultant phase θ_{VR} = 0°, while the corresponding current reflection coefficients are 180° out of phase with the voltage coefficients. The magnitudes of current are equal and their resultant phase θ_{IR} = 180°. Thus, in accordance with the principles described earlier, when resultant voltage and current phases are θ_{VR} = 0° and θ_{IR} = 180°, an open circuit to rearward traveling reflected waves is established and total re-reflection is achieved.

In general, when the equal but opposite phase angles of the two reflected voltages are between 0° and ± 90°, the resultant phases of voltage and current are θ = 0° and 180°, respectively, establishing an open circuit. Conversely, when the equal but opposite voltage phase angles θ are between ± 90° and 180°, the resultant phases of voltage and current are 180° and 0°, respectively, establishing a short circuit.

We'll now examine the reason the phases of the reflected waves change to zero for both voltage and current relative to the source waves after re-reflection. Because the resultant voltage phase at the open circuit is already at 0° prior to re-reflection, there is no change in phase of voltage on re-reflection. Thus the resultant voltage component of the powers re-reflected from both the network output and the antenna mismatch is now traveling forward in phase with the source voltage wave as part of the total forward-traveling voltage wave. Because the resultant current phase is 180° prior to re-reflection but encounters a 180° change in-phase on re-reflection at the open circuit, the resultant current component of the re-reflected powers is now also traveling in phase with the source current wave as part of the total forward-traveling current wave. Consequently, the ultimate result is that all of the power reflected at the load mismatch and transmitted rearward through the network output is totally re-reflected at the network input.

Power Loss Through Use of Steve's Eq 13

Let's now examine Steve's Eq 13, Part 3:

$$P_F = (\sqrt{P_1} + \sqrt{P_2})^2 \qquad \text{(Eq14)}$$

derived from his Eq 9. He used Eq 13 in the section "The T Network Tuner," with the manipulation 75 W + 8.33 W = 133.33 W, with P_1 = 75 W of source power and P_2 = 8.33 W of reflected power. Using this equation led him to overlook 50 W in his power budget, necessary to correct the error in the power equation above. Substituting P_1 and P_2 in Eq 13 does indeed yield 133.33 W, but this answer itself proves the equation invalid. The reason for this error is in incorrectly adding V_1 and V_2 as vectors with phases other than zero. So, why would such an error be made? On discovering that the sum of the in-phase re-reflected and source voltage waves did not yield the correct forward voltage, 81.65 V, he evidently assumed that the two voltage waves must then be added vectorially to obtain the correct 81.65 V. Had he considered that forward voltage and current travel in phase on a lossless line, thus making both V_1 and V_2 travel at zero phase, the analysis likely would have come right.

Continuing the discussion concerning the missing 50 W mentioned above, we'll now examine what happened to the 25 W that reached the network input (at –32.940 + j12.843 V, Steve asserts) after traveling rearward from the network output. This brings us directly to the cause of the

It's a Real Rat Race!

Here is a specific circuit example that may surprise some of you. It proves that a physical discontinuity is not necessary to achieve open and short circuits in transmission lines.

One of the gems of all transmission-line circuitry is the ring diplexer, also known as a hybrid ring or "rat race." See Fig 1. Its apparent magic is traditionally used to isolate two signal sources closely related in frequency that deliver power into dual loads. It uses only wave interference to obtain isolation between the sources. It seems like magic because wave interference is established in the diplexer with no physical discontinuities anywhere. It is an elegant way to drive crossed dipoles to obtain circular polarization (CP) by simply adding baluns at the terminals of the dipoles, plus an additional λ/4 line section in one of the two lines. Right-hand CP is developed from one transmitter and left-hand CP from the other.

Observe that the diplexer comprises nothing more than a continuous loop of transmission line having four terminal ports: two inputs (A and C) and two outputs (B and D). Note that both lines AB and BC on the left-hand side are λ/4 in length, for a total of 180°; on the right-hand side, line AD is l/4 and line DC is 3λ/4, for a total length of 360°. The differential line length between the two sides is therefore 180°, which is the desired number to obtain isolation between the two sources. Here is how it works.

Current induced by voltage applied at A splits at A and travels to C along both sides of the diplexer, through B and D. Because of the 180° difference in length between the two sides, equal voltages arrive at C 180° out of phase, establishing a short circuit at C. The reverse is true also: Current induced by voltage applied at C travels to A in a similar manner, where two equal voltages from C arrive at A 180° out of phase, thus establishing a short circuit at A. Consequently, the short circuit at C prevents current from A from entering the source at C; similarly, the short circuit at A prevents current from C from entering the source at A.

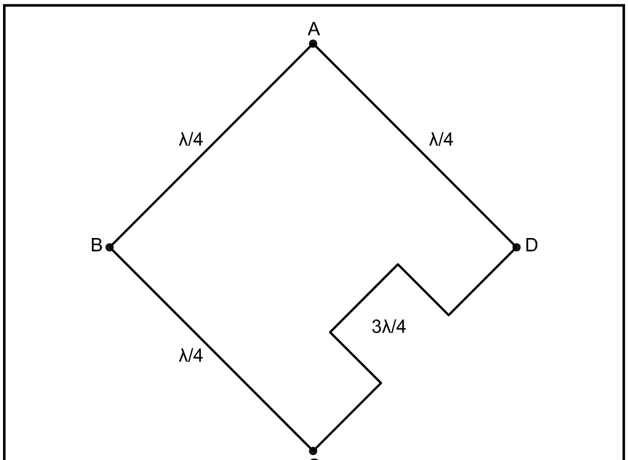

Figure 1—Ring diplexer configuration.

With Z_0 loads placed at B and D, current applied at A will enter both loads, but will not enter lines BC and DC (except in the amount required to overcome resistive loss to maintain the short circuit at C). Similarly, current applied at C will enter both loads but will not enter lines BA and DA (except enough to maintain the short circuit at A). Why? Here's where the apparent magic continues.

Observe that the lengths of all the lines are odd multiples of a quarter wavelength, λ/4 and 3λ/4. Short circuits have been established at both A and C. Terminated in a short, lines whose lengths are odd multiples of λ/4 see an open circuit at their input terminals. Consequently, in traveling to loads at B and D, current from A sees an open circuit looking into lines BC and DC because of the short circuit at C. Similarly, in traveling to loads B and D, current from C sees an open circuit looking into lines BC and DC because of the short circuit at A. Consequently, there are two open circuits looking in opposite directions from both B and D. With the open circuits at B and D between A and C, in tandem with the short circuits at A and C, we have two frequency-dependent sections providing mutual isolation between A and C.

In practice, the degree of isolation depends on the exactness of line lengths and the uniformity of the dielectric constant of the insulating material. By measurements of diplexer circuits I've fabricated in stripline printed-circuit boards, with Teflon/Fibreglass as the insulating material, I've obtained 40 dB of isolation between A and C at high-band VHF.

In systems where the source and load impedances both equal Z_0, the characteristic impedance Z_{0DL} of the diplexer lines must account for the parallel circuitry at the source ports. Therefore,

$$Z_{0DL} = \sqrt{2Z_0^{\,2}} \qquad \text{(Eq 15)}$$

Thus, where Z_0 = 50 ohms, Z_{0DL} = 70.7 ohms, which allows two parallel 100-ohm line inputs at both A and C to present a 50-ohm load to the sources.

So it's not really magic but just simple wave mechanics that establish open and short circuits without any physical discontinuities. Neat, eh? *W2DU* error concerning the missing 50 W. In Steve's example, the power source initially delivers 100 W into impedance Z = 50 + j0 at the input of the line connecting the network to the source. Repeating earlier statements for convenience, on encountering the initial impedance Z = 117.81 – j57.06 ohms at the network input, where, ρ = 0.5, there are 75 W transmitted through the network and 25 W reflected toward the source. Note that the voltage reflected at the network input is 32.94 – j12.843 V, the negative of the voltage he claims arrived rearward from the network output. On reaching the source, the 25 W reflected rearward from the network input creates an initial 3:1 mismatch between the source and the network input, thus reducing the initial source delivery to 75 W. However, on reaching the steady state, the network input impedance has changed to Z_{IN} = 50 + j0; thus the source again delivers 100 W, and the 25 W initially reflected toward the source is now re-reflected in the forward direction at the network input. This is the first 25 W of the missing 50 W that Steve overlooked.

Further, observe that although the values are incorrect, the voltage initially reflected rearward from the network input appears to be the exact negative of the voltage at the input in his model because of the rearward-traveling 25 W returning through the network. Seeing this relationship, he reached the incorrect conclusion because these voltages appeared to cancel and there could be no further rearward travel of voltage into the source line. In the steady state, the 25 W initially reflected rearward from the network input toward the source actually becomes re-reflected forward at the input, and the 25 W that traveled rearward through the network is also re-reflected forward at the input. Those two 25-watt packets of power comprise the 50 W of power overlooked. That is why he considered the source was delivering only 75 W, where he states, *"The fact that the forward source power is 75 W, rather than 100 W, is significant in correctly interpreting the steady-state conditions and the relationship between the total steady-state forward and re-reflected powers."* We earlier proved the above statement to be untrue, and that Steve's Eq 13 is invalid when there is only a single source. We have also proven that in the steady state, the source is delivering 100 W; and we have also found the missing 50 W.

Conclusion and Acknowledgements

To conclude, we have exposed, analyzed, and corrected certain misconceptions, proving that claims that material appearing in *Reflections* is incorrect are unfounded. I want to thank George Baker, W5YR, and James Reid, KH7M for their support on this article.

Notes

[1]S. R. Best, VE9SRB, "Wave Mechanics of Transmission Lines, *QEX,* Part 1, Jan/Feb 2001, Part 2, July/Aug 2001, Part 3, Nov/Dec 2001.

[2]J. C. Slater, *Microwave Transmission,* McGraw-Hill, New York, pp 58-60, 1943, (Chairman, Dept of EE, MIT.)

[3]A. Alford, "Broad-Band Antennas", *Very High-Frequency Techniques*, Radio Research Laboratory, Harvard University, McGraw-Hill, New York, pp 10-15, 1947.

[4]W. Maxwell, W2DU, *Reflections*, Chapter 3, Sec 3.1, 1990.

[5]W. Maxwell, W2DU, *Reflections II*, Chapter 3, Sec 3.1, and Chapter 23, 2001.

[6]W. Maxwell, W2DU, "Examining the Mechanics of Wave Interference in Impedance Matching, *QEX*, Mar/Apr 1998, and Chapter 23, Reflections II.

[7]W. Maxwell, W2DU, "Another Look at Reflections," Part 4, *QST*, October 1973.

[8]W. C. Johnson, *Transmission Lines and Networks,* McGraw-Hill, 1950, Pages 98, 99 and 150, Eq 6.17.

[9]R. W.P. King, Professor of Physics, Harvard University, *Theory of Transmission Lines,* McGraw-Hill, 1955, and Dover Publications, 1965, pp 244 and 464.

Walt's first membership in the ARRL was in 1928, and he has been a Life Member of both the ARRL and QCWA since 1968. He is a Fellow in the Radio Club of America. Licensed as W8KHK in 1933, and continuously licensed since then for 71 years, he held the Advanced Class from 1939 to 1967, when he earned the Extra Class license. Later call signs, W8VJR, W2FCY, and then W2DU in 1968. He was also trustee of the original Boy Scout station, K2BSA. He entered Central Michigan University in 1935 and earned a BS degree in math and physics.

After obtaining 1st Class Radio Telephone and 2nd Class Radio Telegraph licenses, and while baby-sitting BC station WMFJ, Daytona Beach, FL in 1940, with no teletype yet available he copied the news for broadcast from Press Wireless WCX/WJS at 38 wpm.

He engineered antennas at secondary FCC monitoring stations and was a Navy instructor of aviation radio technicians during WW2. Following the war, and a few years as an AM broadcast engineering consultant, he joined the RCA Laboratories, Princeton, NJ, as an engineer in its antenna laboratory. He was a charter engineer at RCA's newly created Laboratories' spinoff, the Astro-Electronic Division, designer and manufacturer of government sponsored spacecraft, also at Princeton.

Walt designed and directed, hands-on, the five VHF ground stations spanning from Cape Canaveral to California, supporting the orbiting Atlas rocket of Project SCORE, carrying the World's first repeater in space. He also designed all antenna systems that flew on the World's first weather satellite, TIROS I, and continued through the VHF antennas on TIROS M (9). Walt invented a 56 dB VHF helical resonator notch filter weighing 7.8 oz, used in TIROS and other satellites. He also designed the antennas that flew on ECHO 1 and 2.

Walt was chief engineer of RCA-Astro's antenna lab until his retirement to Florida in 1980. His 1970's R&D research into more than 1,000 configurations of the quadrifilar helix antenna led to the use of VHF and UHF quadrifilars on all TIROS-N and all now flying as NOAA polar-orbiting weather satellites. His R&D report on the quadrifilar helix antenna research can be viewed in his book, Reflections—Transmission Lines and Antennas II, and can also be downloaded from his web page at **http://home.iag.net/~w2du**.

Walt's hobbies include experimenting in his own well-equipped RF lab, technical writing, photography, performing on string bass in big bands and small jazz groups, Florida boating, and global cruising with his wife, Jean. □□